Artículo de investigación

Relationship between lacunarity and bandwidth of a koch-type wire antenna

Relación entre la lagunaridad y el ancho de banda de una antena de cables de tipo koch

Relação entre lacunaridade e largura de banda de uma antena de arame koch-type

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Abstract

A dipole wire antenna of the Koch type is considered. The antenna represents a wire dipole symmetrical with respect to the point of feeding. Arms of the dipole have a geometry similar to Koch's pre-fractal. The curves forming the arms differ from the classical Koch fractal only by the position of the central vertex. A family of antennas is singled out, in which the antennas differ from each other by coordinates of the central vertices. An algorithm for calculating lacunarity is described. A correlation analysis is provided with a correlation of bandwidth as well as relative bandwidth with lacunarity. Antennas having the geometry of the first three iterations of a Koch-type curve are chosen for the analysis. The calculated correlation coefficients are given in the tables. It is shown that increasing the iteration leads to a decrease in the correlation between the selected parameters. It is obtained that the correlation coefficients for the relative bandwidth are smaller than those for the bandwidth. Single-parameter regression models for the bandwidth and the relative bandwidth are constructed. The root-mean-square errors for the models are calculated. The proposed regression formulas can be used to design broadband wire antennas.

Keywords: Koch-type antenna, bandwidth, fraction bandwidth, lacunarity, regression analysis.

Resumen

Se considera una antena de cable dipolo del tipo Koch. La antena representa un dipolo de alambre simétrico con respecto al punto de alimentación. Los brazos del dipolo tienen una geometría similar al prefractal de Koch. Las curvas que forman los brazos difieren del fractal clásico de Koch solo por la posición del vértice central. Se selecciona una familia de antenas, en la que las antenas difieren entre sí por las coordenadas de los vértices centrales. Se describe un algoritmo para calcular la lagunaridad. Se proporciona un análisis de correlación para la correlación del ancho de banda, así como el ancho de banda relativo con lagunaridad. Las antenas que tienen la geometría de las tres primeras iteraciones de una curva de tipo Koch se eligen para el análisis. Los coeficientes de correlación calculados se dan en las tablas. Se muestra que aumentar la iteración conduce a una disminución en la correlación entre los parámetros seleccionados. Se obtiene que los coeficientes de correlación para el ancho de banda relativo son más pequeños que los del ancho de banda. Se construyen modelos de regresión de un solo parámetro para el ancho de banda y el ancho de banda relativo. Se calculan los errores de raíz cuadrada-media para los modelos. Las fórmulas de regresión propuestas pueden usarse para diseñar antenas de cable de banda ancha.

Palabras claves: Antena de tipo Koch, ancho de banda, ancho de banda de fracción, lagunaridad, análisis de regresión.

Resumo

É considerada uma antena de cabo dipolo do tipo Koch. A antena representa um dipolo de arame simétrico em relação ao ponto de alimentação. Os braços do dipolo têm uma geometria similar à prefractal de Koch. As curvas que formam os braços diferem do fractal clássico de Koch apenas devido à posição do vértice central. Uma família de antenas é selecionada, na qual as antenas diferem umas das outras pelas coordenadas dos vértices centrais. Um algoritmo é descrito para calcular a lacunaridade. Uma análise de correlação é fornecida para a correlação da largura de banda, bem como a largura de banda relativa com a lagunaridade. As antenas que possuem a geometria das três primeiras iterações de uma curva do tipo Koch são escolhidas para a análise. Os coeficientes de correlação calculados são fornecidos nas tabelas. É mostrado que o aumento da iteração leva a uma diminuição na correlação entre os parâmetros selecionados. Obtém-se que os coeficientes de correlação para a largura de banda relativa são menores que os da largura de banda. Modelos de regressão de um único parâmetro são construídos para a largura de banda e a largura de banda relativa. Os erros médios de raiz quadrada para os modelos são calculados. As fórmulas de regressão propostas podem ser usadas para projetar antenas de cabo de banda larga.

Palavras-chave: Antena Koch, largura de banda, fração de banda, lagunaridade, análise de regressão.

Introduction

Wire antennas are widely used in modern telecommunication systems (Balanis, 1997). However, if the simplest wire half-wave dipole antenna is already well-understood, then the antennas with a more complex geometry represent a separate subject for investigation (Poole, 2011). It is the antennas with a complex geometry that are by far the most promising devices. It is so because, by complicating the geometry, one can both minimize the dimensions of the antenna itself and improve its electrodynamic characteristics (Singh, 2009).

Various shapes of broken symmetrical dipoles are used in practice (Balanis, 1997; Nasr, 2013; Milligan, 2005). But the most common way to minimize or improve the given properties of antennas is their fractalization (Gianvittorio and Rahmat-Samii, 2002; Baker and Iskander, 2015; Karpukov et al, 2002; Wagh, 2015; Krzysztofik, 2013; Beigi and Mohammadi, 2016). The most researched fractal antenna is a dipole, constructed on the basis of the Koch prefractal. A sufficient number of works are devoted to the study and analysis of the main characteristics of both the Koch dipole and the Koch monopole and its various modifications (Baliarda, 2000; Li, 2012; Rani. 2012; Vinoy, 2014; Karim, 2010). For the simplest half-wave dipole, especially for the dipole of the base frequency, the connections between its various parameters are well-known (Banerjee and Bezboruah, 2014).

Various methods are used for the analysis and synthesis of antennas with complex geometry. For example, in Surutka and Velickivica method for investigating wire dipoles is presented, leading to a system of two integral equations that are solved approximately by the method of coincidence of points (Surutka and Velickivic, 2003). In Li, the dipole constructed on the basis of the Koch prefractal was investigated by the moment method (Li et al, 2012). In Tumakov, a correlation-regression analysis was proposed for establishing relationships between the main electrical characteristics and geometric parameters of the antenna (Tumakov et al, 2017). The main approach to antenna analysis is the study of the influence of the radiator geometry on the antenna characteristics, as it can be seen, for example, in (Abgaryan et al, 2017; Markina et al, 2017; Markina et al, 2017; Markina et al, 2018).

The above analysis methods apply to any antennas. In the study of fractal antennas, the lacunarity can be taken into account (Sengupta and Vinoy, 2006). For example, the effect of the lacunarity on the base frequency of a Koch-type dipole is considered in (Abgaryan and Tumakov, 2017). In the present paper, we analyze the interrelationships of the bandwidth with the lacunarity for the Koch-type wire dipole. A correlation analysis of the bandwidth for different values of the cutoff of the reflection coefficient and the lacunarity is carried out. Regression formulas for the bandwidth are obtained in the case where the antenna is formed by a Koch-type prefractal of the first three iterations.

Prefractals of the Koch Type

As is known, the fractal Koch curve can be constructed using the following iterative scheme (Barnsley and Harrington, 1989):

$$\boldsymbol{K_0} = [\boldsymbol{0}, \boldsymbol{1}], \qquad \boldsymbol{K_n} = \bigcup_{i=1}^{4} A_i(\boldsymbol{K_{n-1}}), \qquad \boldsymbol{K} = \lim_{n\to\infty} \boldsymbol{K_n}, \tag{1}$$

where $A_i, i = 1..4$ are the affine transformations of the plane:

$$A_1\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{3}\begin{pmatrix}x\\y\end{pmatrix},$$

$$A_2\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1/3\\0\end{pmatrix},$$

$$A_3\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1/2 & \sqrt{3}/2\\ -\sqrt{3}/2 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1/2\\1/2\sqrt{3}\end{pmatrix},$$

$$A_4\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{3}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}2/3\\0\end{pmatrix}$$

The generating set K_0 and the first three iterations of the Koch curve are shown in Fig. 1.

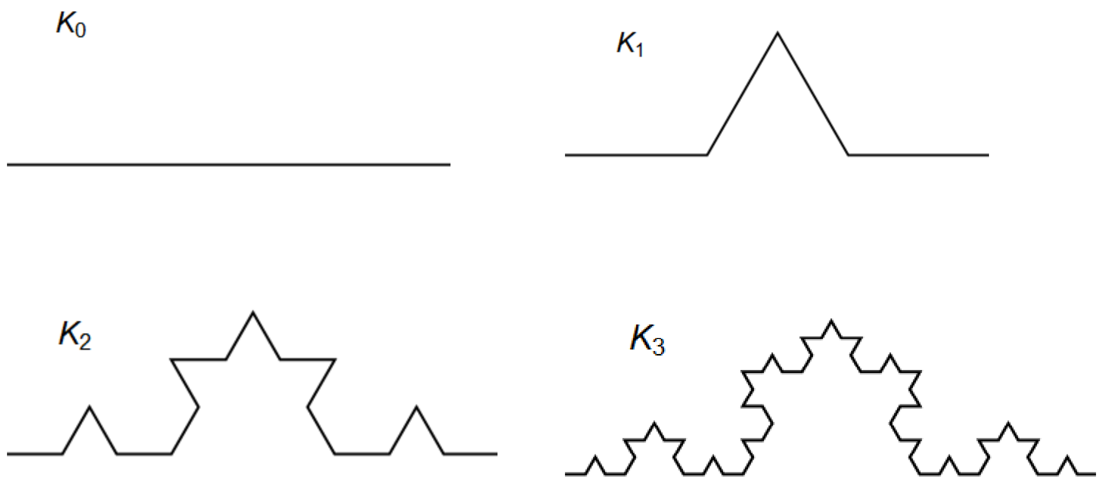


Figure 1. The first iterations of the Koch curve

In fractal theory, the set K_1, as a rule, is called the Koch fractal generator. It is obvious that the Koch generator can be defined as a line connecting five points (vertices): $\{x_0 = (0,0),\ x_1 = (0.333, 0), x_2 = (0.5, 0.288), x_3 = (0.666, 0), x_4 = (1,0)\}$. Suppose that these points are given arbitrarily; then for constructing the fractal curve generated by the generator, we use the algorithm of fractal interpolation (Igudesman et al, 2015)

Let us describe this algorithm in more detail. Let

$$\boldsymbol{K_1} = \left((\boldsymbol{t_i}, \boldsymbol{x_i}) \in [\boldsymbol{0}, \boldsymbol{1}] \times \mathbb{R}^2 \,\middle|\, \boldsymbol{0} < \boldsymbol{t_0} < \cdots < \boldsymbol{t_4} = \boldsymbol{1}\right) \tag{2}$$

be the interpolation points. We define affine transformation of the space as follows:

$$\boldsymbol{A_i}\colon \mathbb{R}^3 \to \mathbb{R}^3,\ \boldsymbol{A_i}\begin{pmatrix}\boldsymbol{t}\\ \boldsymbol{x}\end{pmatrix} = \begin{pmatrix}\boldsymbol{a_i} & \boldsymbol{0}\\ \boldsymbol{c_i} & \boldsymbol{D_i}\end{pmatrix}\begin{pmatrix}\boldsymbol{t}\\ \boldsymbol{x}\end{pmatrix} + \begin{pmatrix}\boldsymbol{e_i}\\ \boldsymbol{f_i}\end{pmatrix}, \tag{3}$$

where a_i, c_i, e_i, f_i are unknown parameters, and the matrix D_i is parameter transformation. Next, we require that for all $i = 1, \dots, 4$ the following two conditions are to be satisfied:

$$\boldsymbol{A_i}\begin{pmatrix}\boldsymbol{t_0}\\ \boldsymbol{x_0}\end{pmatrix} = \begin{pmatrix}\boldsymbol{t_{i-1}}\\ \boldsymbol{x_{i-1}}\end{pmatrix},\ \boldsymbol{A_i}\begin{pmatrix}\boldsymbol{t_4}\\ \boldsymbol{x_4}\end{pmatrix} = \begin{pmatrix}\boldsymbol{t_4}\\ \boldsymbol{x_4}\end{pmatrix}. \tag{4}$$

Then

$$\boldsymbol{a_i = t_i - t_{i-1},\ e_i = t_{i-1},\ c_i = x_i - x_{i-1} - D_i(x_4 - x_0),\ f_i = x_{i-1} - D_i x_0}. \tag{5}$$

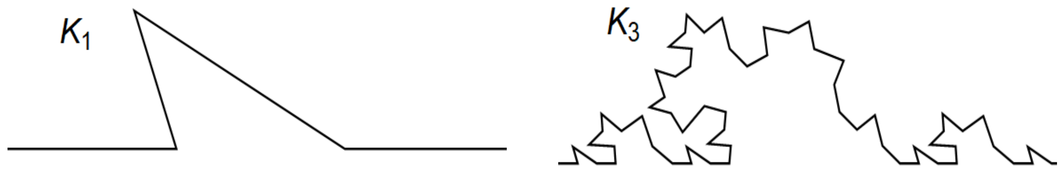
As a result, all unknown quantities have been found, and for constructing a fractal curve, one can use the standard iterative scheme (1).

As an example, let us construct with the help of the described algorithm prefractals for interpolation points $K_1 = \{x_0 = (0,0),\ x_1 = (0.333, 0),\ x_2 = (0.25, 0.25),$ $x_3 = (0.666, 0),\ x_4 = (1, 0)\}$.

We set the parameters of the scheme as follows:

$$t_i = 0.25 + t_{i-1}, i = 1, \dots, 4,$$
$$D_1 = D_4 = \tfrac{1}{3}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\ D_2 = \tfrac{1}{3}\begin{pmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{pmatrix}, D_3 = \tfrac{1}{3}\begin{pmatrix}1/2 & \sqrt{3}/2\\ -\sqrt{3}/2 & 1/2\end{pmatrix}.$$

As a result of fractal interpolation, a curve similar to the Koch curve is obtained (Fig. 2).



(a) (b)

Figure 2. The first (a) and third (b) iterations of a Koch type curve.

We construct the set of Koch-type curves with coordinates of the median vertex that varies in x from 0.035 to 0.75 in steps of 0.035 and in y from 0.035 to 0.8 in steps of 0.035. We obtain 440 different curves that initiate the fractal.

Lacunatity

The concept of the lacunarity was introduced by Benoit Mandelbrot in 1977. Despite the fact that the Latin lacuna is translated as the vacuum, in the generalized sense, the lacunarity should be understood as the measure of the inhomogeneity of the set. The definition of this concept is inseparable from the description of the calculation algorithm, so we will define it by describing the Gliding Box algorithm. The computer algorithm Gliding Box usually relies on the representation of a given set in the form of a matrix of pixels (Plotnick et al, 1996).

Let A be a set from a two-dimensional $\mathbb{R}^2$ space. We restrict the given set in the plane to a rectangle of optimal dimensions, then such a construction can uniquely match the matrix of Lac pixels of dimension $M \times L$, depending on the size of the bounding box. We fix the box size of r; under the box mass (m) we mean the number of unused (empty) pixels in the image set.

The implementation of the algorithm can be imagined as a single "sliding" rectangle box that reads the mass in each position, or, as an ordered coverage of a rectangle by a multitude of boxes. The quantities dx, dy, which define the distance between two adjacent boxes in one row and in one column, respectively, will be considered as the coverage parameters. Fig. 3 shows an example of covering the Koch curve of the third iteration with an image dimension $M \times L = 25 \times 75$ pixels.

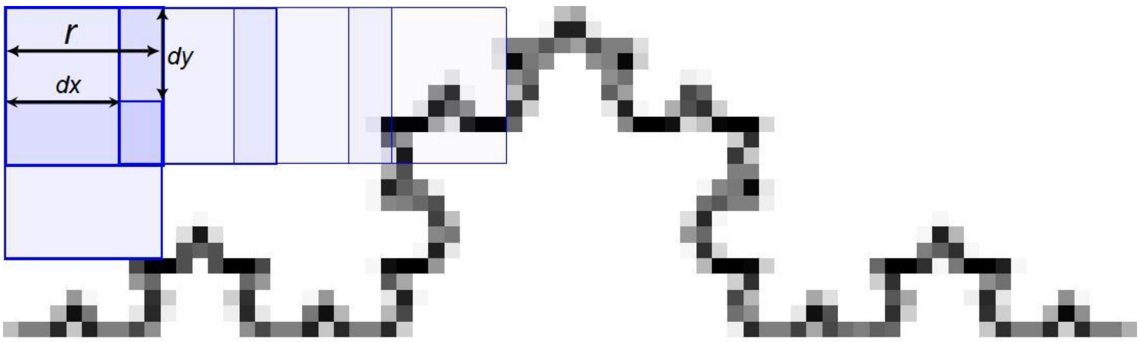


Figure 3. Covering of the pixel image of the Koch curve of the third iteration such image can uniquely match the matrix.

$$P(m,r) = n(m,r)/\sum_{m=1}^{r^2} n(m,r), \tag{7}$$

and the second moment, respectively

$$M_1 = \sum_{m=1}^{r^2} m\,P(m,r),\ \ M_2 = \sum_{m=1}^{r^2} m^2 P(m,r).$$

Transformation of the last expressions gives:

$$M_1 = E(m), \qquad M_2 = E(m^2) = Var(m) + E^2(m), \tag{8}$$

where E and Var are, respectively, the mathematical expectation and the variance. The lacunarity is defined as

$$\Lambda(r) = M_2/[M_1]^2. \tag{9}$$

Taking into account the relation $Var(m) = \sigma^2(m)$, where $\sigma(m)$ is the standard deviation, expression (9) can be reduced to the form $\Lambda(r) = V^2 + 1$, where V is the variation. The lacunarity, thus, is a measure of the relative mass spread m; more precisely, it shows what fraction of the average value of this quantity is its average spread. Since the lacunarity value depends on the box size, calculations should be made for a number of boxes with different sizes. Finally, the conclusion about the lacunarity of the set can be made by analyzing the dependence of Λ on the box size r.

We calculate the lacunarity of Koch prefractal of the third iteration using the Gliding Box algorithm. For convenience of further calculation, we limit the curve so that the dimension of the matrix is $Dim(Lac) = M \times L = 400 \times 1200$ (Fig. 4).

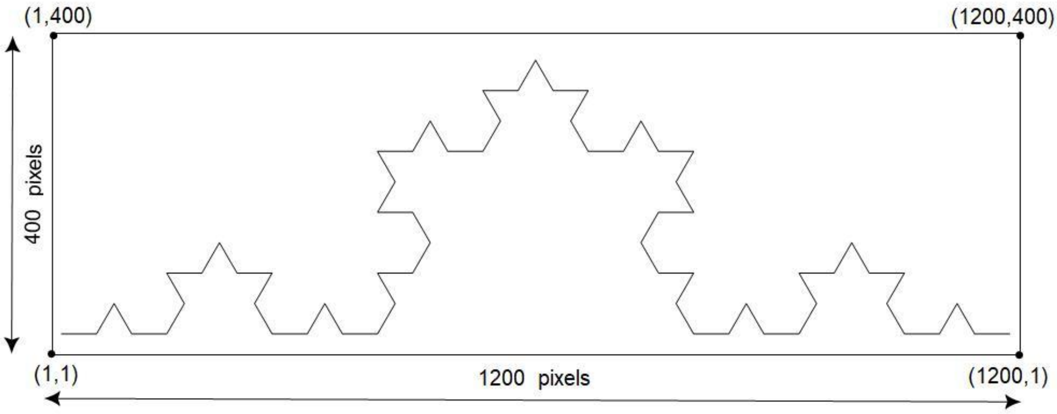


Figure 4. Pixel image of the Koch curve

Moreover, the coordinates of the ends of the segment $[0, 1] \in \mathbb{R}$, which represents the initiating segment for the Koch curve, will have the coordinates $(6, 323)$ and $(1194, 373)$, respectively, in the matrix Lac, and the vertex of the initiating triangle will be $(600, 27)$. The calculations are carried out for box sizes that are common integer divisors of the dimension of the matrix Lac; this choice is related to the fact that it is possible to cover the matrix with an integer number of boxes.

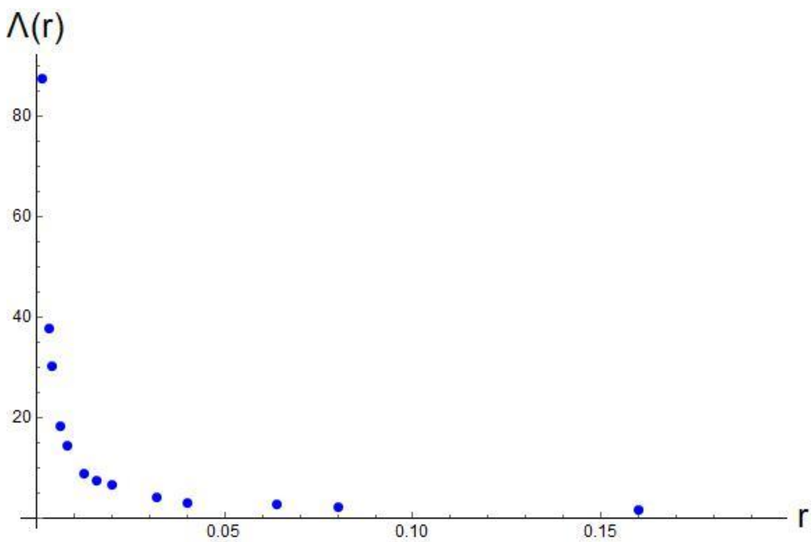


Figure 5. Dependence of lacunarity on the size of the box

The results of calculating the lacunarity values for the Koch curve from the box size for the values of the parameters $dx = dy = r$ are shown in Fig. 5. With this choice of r, the covering degenerates into a grid of nonintersecting boxes.

Correlation Analysis

We determine the relationship between the parameters of the dipole using the selective correlation coefficient r_{XY}, which is calculated by the formula

$$\boldsymbol{r_{XY}} = \frac{\sum_{i=1}^{n} \boldsymbol{X_i Y_i} - \boldsymbol{n}\,\overline{\boldsymbol{X}}\,\overline{\boldsymbol{Y}}}{\boldsymbol{n}\,\widehat{\boldsymbol{\sigma}}_{\boldsymbol{X}}\,\widehat{\boldsymbol{\sigma}}_{\boldsymbol{Y}}}, \quad (10)$$

where n is sample size; $\bar{X}$ and $\hat{\sigma}_X$ are sample mean and sample mean deviation, respectively. It can be shown that $0 \leq |r_{XY}| \leq 1$; here, if $|r_{XY}| = 0$, then the quantities are independent, and if $|r_{XY}| = 1$, then they are linearly dependent. Thus, r_{XY} serves as a measure of the linear relationship between samples X and Y.

Let us take the scale for antenna design equal to 7.5 cm. Then we obtain a set of 440 antennas (one antenna for each iteration) with a central vertex whose coordinates vary within the following limits: the x coordinate varies from 2.625 mm to 5.625 cm in 2.625 mm increments; the y coordinate varies from 2.625 cm to 6.0 cm in 2.625 mm increments. Now, for the correlation analysis, we choose the lacunarity Λ as one parameter, and as the other parameter, we choose the bandwidth BW for $S_{11} < -3\ dB$, $S_{11} < -5\ dB$ and $S_{11} < -10\ dB$. We determine the coefficients of pair correlation by formula (10) for the selected parameters. The values obtained are given in Table 1.

Table 1. Values of pair correlation coefficients between bandwidth and lacunarity

	$\boldsymbol{S_{11}} < -3\ dB$	$\boldsymbol{S_{11}} < -5\ dB$	$\boldsymbol{S_{11}} < -10\ dB$
first iteration	-0.965	-0.953	-0.895
second iteration	-0.918	-0.894	-0.899
third iteration	-0.863	-0.863	-0.712

From the results given in Table 1, one can conclude that there is a general tendency to reducing the relationship with an increase in the iteration of the prefractal. However, for all the cases considered, the relationship between the lacunarity and the bandwidth is strong. As it was expected, the correlation coefficient decreases as the allowable values of S_{11} decrease, since the number of antennas having zero bandwidth is increased.

Now let us consider the relationship between the relative bandwidth of $B = BW/f$ and the lacunarity. Table 2 shows the obtained values of the correlation coefficients. The comparison of two tables shows that in most cases the correlation coefficients for B are slightly smaller than those for BW. A similar phenomenon also occurs for the correlation of the bandwidth and the length of the dipole conductor (Tumakov et al, 2017). Note that for the third iteration the values of the correlation coefficients are the same.

Table 2. Values of pair correlation coefficients between relative bandwidth and lacunarity

	$\boldsymbol{S_{11}} < -3\ dB$	$\boldsymbol{S_{11}} < -5\ dB$	$\boldsymbol{S_{11}} < -10\ dB$
first iteration	-0.937	-0.906	-0.833
second iteration	-0.885	-0.827	-0.876
third iteration	-0.863	-0.843	-0.719

Negative values of the correlation coefficients indicate the fact that an increase in the lacunarity leads to a decrease in the bandwidth. It factually means that the inhomogeneity of filling the space with the antenna wire (chaotic arrangement of the arm elements) results in a narrow passband.

In the following paragraphs, we construct regression models for the case of $S_{11} < -10\ dB$. We begin the construction of models with the bandwidth.

Bandwidth

The linear relationship between the bandwidth and the lacunarity for the first-iteration Koch dipole is strong ($r_{XY} = -0.895$). The analysis of the correlation field (Fig. 6) shows that it is better to choose the regression model as a polynomial of the second degree:

$$\widehat{BW} = -1205.91 + 1277.06\,\Lambda - 317.62\,\Lambda^2. \quad (11)$$

For the model, we calculate the absolute error ε by the formula

$$\varepsilon = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}(X_i)\right)^2},$$

where Y_i are the exact values; $\hat{Y}(X_i)$ are the values determined by the regression formula at the points X_i. We obtain $\varepsilon = 9.85$ MHz.

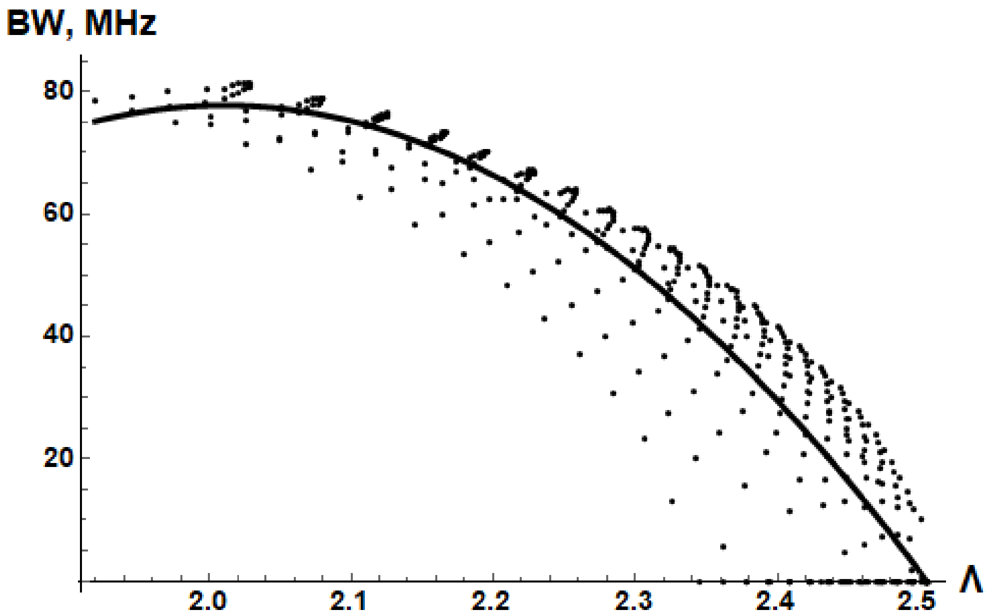
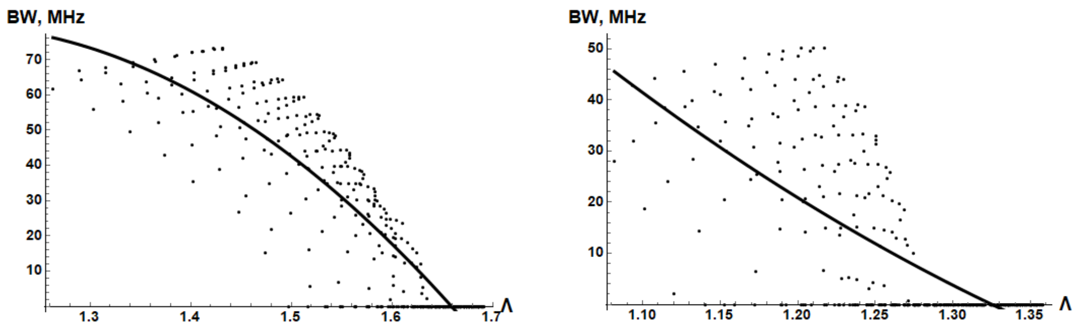


Figure 6. Regression of BW on Λ for the first iteration Koch's dipoles

Let us now consider the relationship between the bandwidth and the lacunarity values for the higher iterations. Fig. 7(a) and 7(b) show the correlation fields for the second and third iterations, respectively. Apparently, the scatter of points increases with the increase in iteration. Moreover, the points are located practically along the entire correlation field for the third iteration.



(a) (b)

Figure 7. Regression of BW on Λ for dipoles of the second (a) and third (b) iterations

Nevertheless, we construct models for the second iteration:

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$$\widehat{BW} = -344.881 + 733.732\,\Lambda - 316.957\Lambda^2 \quad (12)$$

with the error $\varepsilon = 11.16$ MHz and for the third iteration

$$\widehat{BW} = 495.244 - 602.171\,\Lambda + 172.386\Lambda^2, \quad (13)$$

with the error $\varepsilon = 10.278$ MHz.

As one can see from Fig. 7, with an increase in the prefractal iteration the regression model approximates the structure of the correlation field worse and worse. We also note that an increase in the prefractal leads to an increase in the number of dipoles with a zero bandwidth. Among the first iteration dipoles, 73 out of the 440 antennas have a zero bandwidth, and for the second and third iterations, the numbers are 217 and 316 antennas, respectively.

Fractional Bandwidth

Now, let us construct regression models for the relative bandwidth as a function of the lacunarity. Fig. 8 shows the correlation field for B and Λ.

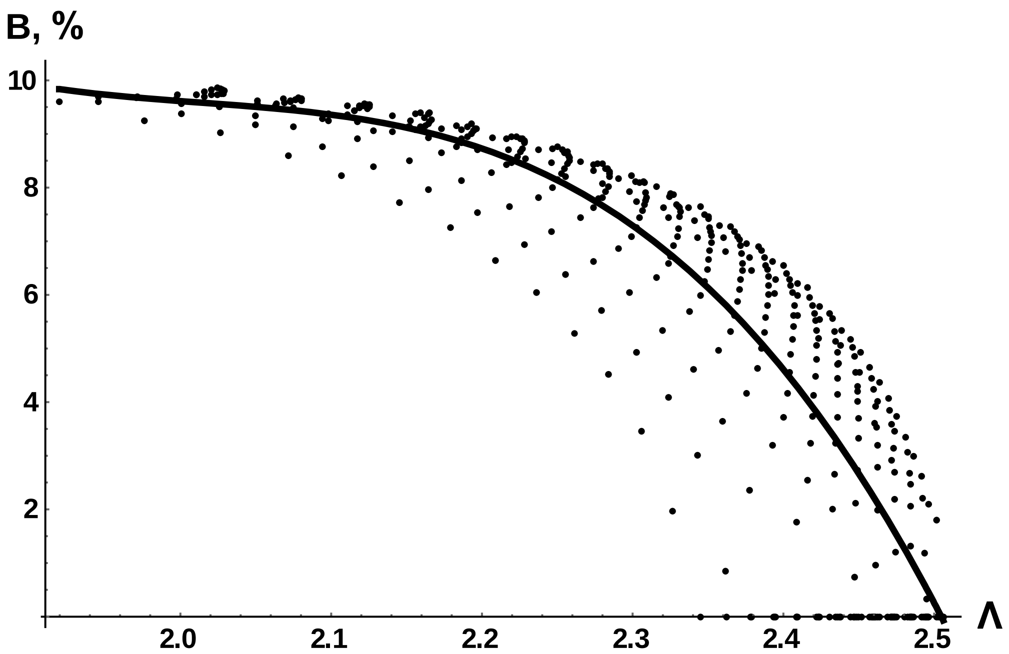

Figure 8. Regression B on Λ for Koch first-iteration dipoles

The regression model is constructed in the form of a polynomial of the third degree:

$$\hat{B} = 602.63 - 879.63\,\Lambda + 435.92\,\Lambda^2 - 72.178\,\Lambda^3. \quad (14)$$

The proposed model has the approximation error $\varepsilon = 1.5\ \%$

We construct models for higher iterations in the same way as in the previous section in the form of polynomials of the second degree. The graphs for the regression curves obtained in the case of the second and third iterations Koch dipoles and correlation fields are shown in Fig. 9(a) and 9(b).

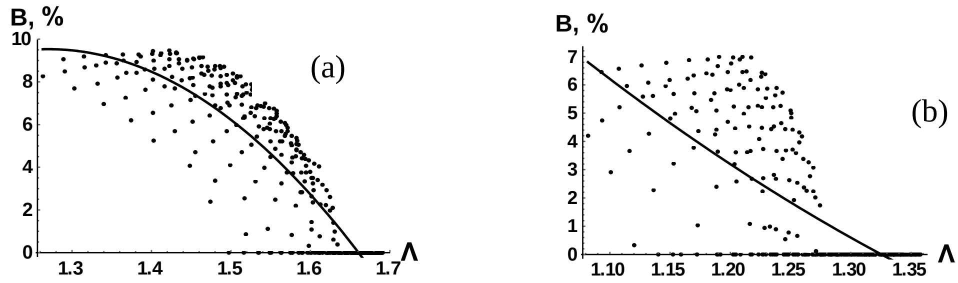

Figure 9. Regression B on Λ for dipoles of the second (a) and third (b) iterations

The regression model for the second iteration can also be constructed of the second order, as for BW:

$$\widehat{\boldsymbol{B}} = \mathbf{-91.13 + 158.54\,\Lambda - 62.425\,\Lambda^2}. \tag{15}$$

The proposed model has the approximation error $\varepsilon = 1.60$ %.

The regression model for the third iteration takes the following form:

$$\widehat{\boldsymbol{B}} = \mathbf{66.75 - 77.86\,\Lambda + 20.75\,\Lambda^2}. \tag{16}$$

This model has the approximation error $\varepsilon = 1.53$ %.

Summary

The relationship of the bandwidth for different levels of the reflection coefficient and the lacunarity on a sample of 440 Koch wire dipoles for each iteration is analyzed. It is shown that the bandwidth is strongly dependent on the lacunarity. It was also concluded that with increasing the iteration of the prefractal forming the dipole, this dependence decreases. Regression models are constructed for the bandwidth and the relative bandwidth in the form of polynomials of the second and third degrees.

It is concluded that the large compactness of the antenna (closely spaced elements of the arm) results in a narrow bandwidth.

Conclusions

The obtained regression models for the bandwidth are of sufficiently high accuracy and can be used in the simulation of wire Koch-type dipoles. The best dependence of the bandwidth on the lacunarity takes place for a dipole constructed on the basis of prefractal of the first and second iterations. Such dipoles have a 10% bandwidth at lacunarity values of $2.0 < \Lambda < 2.3$ and $1.4 < \Lambda < 1.5$, respectively.

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