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Reconstruction of digital filter parameters when changing the data arrival period

Перестройка параметров цифрового фильтра при изменении периода поступления данных

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Abstract

We consider the problem of forming an algorithm for the operation of a digital filter that provides processing, according to a certain law, of discrete samples x[k] of some continuous signal x(t) at the moments of quantization tк=k ·T0, where T0 -[second]- is the discreteness period in time, and k=0,1,2,.. is the integer variable defining dimensionless discrete time. This work poses and solves the problem of forming the digital filter parameters restructuring, which ensures that the filtering properties remain unchanged when the frequency of information is changed, in particular, the constancy of the frequency and pseudo-frequency characteristics of the filter. An algorithm for restructuring the numerical parameters of the filter based on information about the time intervals of information arrival has been developed. At the stage of filter development, a special conversion matrix is formed for the specified parameters, and at the stage of filter operation in real time, an operational recalculation of the digital filter parameters is performed. For the test example, the calculation results are given, showing good tuning accuracy and stable filter characteristics with a significant change in the quantization frequency.

Keywords: digital filters, sampling period, parameter tuning, z-transfer functions, real frequency, pseudo-frequency, frequency response.

Аннотация

Рассматривается задача формирования алгоритма работы цифрового фильтра, обеспечивающего обработку, по определённому закону, дискретных выборок x[k] некоторого непрерывного сигнала x(t) в моменты квантования tк=k ·T0, где T0 -[секунд]- период дискретности по времени, k=0,1,2,.. целочисленная переменная, определяющая безразмерное дискретное время. В данной работе ставится и решается задача формирования перестройки параметров цифрового фильтра, обеспечивающей, при изменении частоты поступления информации, неизменность фильтрующих свойств. в частности постоянство частотной и псевдочастотной характеристик фильтра. Разработан алгоритм перестройки числовых параметров фильтра по информации об временных интервалах поступления информации. На этапе разработки фильтра, для задаваемых параметров производится формирование специальной матрицы пересчёта, а на этапе работы фильтра в реальном времени выполняется оперативный пересчёт параметров цифрового фильтра. Для тестового примера приводятся результаты расчёта, показывающие хорошую точность перестройки и стабильные характеристики фильтра при существенном изменении частоты квантования.

Ключевые слова: цифровые фильтры, период дискретности, перестройка параметров, z- передаточные функции, реальная частота, псевдочастота, частотная характеристика.

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Introduction

In digital control of technology, radio communication and automation, digital filters (DF) are widely used, which provide processing of discrete samples of a continuous signal *x(t)* at fixed times *t=k*T0*, where *T0* (second) is the certain step of discreteness in time, *k=0,1,2..* is the integer variable that defines the dimensionless discrete time. The implementation of such devices is usually carried out on circuits containing registers and adders with certain coefficients; implementation in the form of a digital microprocessor device is possible (Serrezuela et al., 2017; Chen et al., 2020).

A certain filter functional is defined by a specific mathematical model, difference equation, by a corresponding *z*-transfer function, or by a discrete impulse transient function of the filter (Shamrikov, 1985; Oppenheim & Schafer, R. 2018; Antonio, 1983). It should be noted that these models determine the operation of the filter at a specific value of the step *T0=1/f0*, where *f0* (hertz) is the frequency of periodic sample arrivals of *x[k]* continuous signal *x(t)* (Gadzikovsky, 2007; Adamou-Mitiche & Mitiche, 2019).

The time intervals of signal arrival in many technical problems can change significantly during operation, for example, as it is described in (Psiaki et al., 2014; Daneshmand et al., 2015). In particular, this may be due to a change in the nature of transmission over the radio path (Velikanova & Voroshilin, 2012; Xu et al., 2016) and also due to the conditions for receiving and transmitting information from continuous objects. So, in works (Sokolov et al., 2018; Kalmykov et al., 2020) the changes associated with the satellite measurement system are considered.

Without dwelling on the reasons for these phenomena, we will assume that the discreteness period *T0=1/f0* during system operation can change to the value *TN=N/f0*, where *N* is some number characterizing the multiplicity of changes in the period relative to the calculated one (Koshita et al., 2017).

If the frequency changes during operation *f0* of data arrival on frequency *fN=f0/N* the properties of the filter will change, and with a significant change in frequency, such filtering can lead to unsatisfactory results, if the device parameters are not rebuilt. Particularly critical change of *f0* is for digital automatic systems (Shamrikov, 1985), where DF operates in a closed control loop, and a change in their properties can lead to a decrease in stability margins, and, possibly, to a loss of stability of the digital system. We show it on the example of a linear digital filter of the *n*-th order.

Let the processing of a continuous signal *x(t)* by a discrete sample *x[k]* is performed by a linear DF determined by *z*-transfer function of the following form:

$$D0(z) = \frac{A(z^{-1})}{B(z^{-1})} = \frac{a_0 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \cdots + a_{n-1} \cdot z^{-n+1} + a_n \cdot z^{-n}}{1 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + b_3 \cdot z^{n-3} + \cdots + b_{n-1} \cdot z^{-n+1} + b_n \cdot z^{-n}}, \qquad (1)$$

where $z = e^{s \cdot To}$, s is the Laplace transform parameter, $A(z^{-1})$, $B(z^{-1})$ are the polynomials from z^{-1}, determined by a specific set of coefficients a_i, b_i, *i=0..n.* DF of the described type, operating at a frequency *f0 =1/T0,* will be called the reference DF. The corresponding difference equation written with respect to the input *y[k]* and output *x[k]* of DF for the periods of time $t = k \cdot T0$ will take the following form:

$$x[k] = -b_1 \cdot x[k-1] - b_2 \cdot x[k-2] - \cdots - b_n \cdot x[k-n] + a_0 \cdot y[k] + a_1 \cdot y[k-1] + .. + a_{n-1} \cdot y[k-n+1] + a_n \cdot y[k-n]. \qquad (2)$$

This difference equation defines one of the ways to implement DF in the form of an algorithm for calculating the output coordinate *x[k]* by past output *x[k-i]* and input *y[k-i]* values detainees, i.e. saved in memory at the previous steps of calculation. For fixed frequency *f0* DF with *z*-transfer function (1) performs a certain transformation of the spectrum of a continuous signal, for example, it filters out low-frequency or high-frequency signal components or components of a certain frequency, corrects the phase characteristic, etc. (Antonio, 1983; Oppenheim & Schafer, 2018). In this case, Fourier transform of the signal at the output of DF is determined by the following expression:

$$Y(e^{j \cdot \omega \cdot To}) = D0(e^{j \cdot \omega \cdot To}) * X(e^{j \cdot \omega \cdot To}), \qquad (3)$$

where $X = (e^{j \cdot \omega \cdot To})$ is the sample conversion *x[k]* of continuous signal *x(t).*

It should be noted that due to the specific properties of the transcendental function $e^{j\cdot\omega\cdot To}$, spectrum $X(e^{j\cdot\omega\cdot To})$ of discrete signal *x[k]* is periodic with a period $\omega o(\text{radians/second}) = 2 * \pi * f0$ (Shamrikov, 1985).

The samples are received not with the calculated interval *T0*, but with a changed one: *TN=T0*N*, where the parameter *N* determines the multiplicity of the change in the quantization period *TN* relative to the calculated period *T0.* An important point is that the new quantization period *T0*N* should also ensure the transmission of the properties of a continuous signal by discrete samples, which, according to the Nyquist–Shannon theorem, corresponds to the relation $\omega m < \frac{\pi}{T0*N}$, where $\omega m(radians/second)$ is the maximum frequency present in the spectrum of a continuous signal. When changing the quantization period *TN=T0*N*, the frequency response (FR) of DF will also change, which is determined by the following formula:

$$DN(j\cdot\omega, TN) = D0(e^{j\cdot\omega\cdot To*N}) = \frac{A(e^{-j\cdot\omega\cdot To*N})}{B(e^{-j\cdot\omega\cdot To*N})}, \qquad (4)$$

This FR has properties that differ from the properties of the reference filter: it is scaled along the frequency axis, and most importantly, it has a repetition period $2\pi\cdot f0/N$, which is *N* times less (if *N>1*), or more than the original one. For this reason, to choose a "similar filter", i.e. some DF *DN(z,T0· N*) of the same order, which would completely repeat the properties of the reference DF with a change in frequency *f0* to *fN*=f0$/N$, is impossible. With regard to the simplest DF of 1-2 orders of magnitude, a possible approximate approach for generating a conversion is given in (Belonogov, 2014; Kaplun et al., 2020).

In this paper, we pose the problem of calculating the algorithms for tuning the parameters of such a "similar" DF operating at the frequency *fN*, which would have closeness of its FR to the characteristics of the reference filter $D0(e^{j\cdot\omega\cdot To})$ in the baseband of a digital system: $0 < \omega < \pi\cdot f0/N$.

The procedure should ensure the constancy of the dynamic characteristics of DF, while allowing the use of algorithms in the operational restructuring of DF parameters when the period *T0* changes.

On the basis of studies carried out in the frequency domain using the methods of w-transformation, an algorithm for the operational restructuring of the numerical parameters of the filter based on information about the time intervals of information receipt has been developed. A method for calculating the restructuring of parameters based on the preliminary formation of a special recalculation matrix is proposed. At the stage of the filter operation in real time, this matrix implements the operational recalculation of the filter parameters by a linear relation. The proposed approaches make it possible to ensure the constancy of the frequency properties of the digital filter with high accuracy with a significant change in the periods of information receipt. At the same time, the implementation of the proposed approaches does not require significant computing resources and can be carried out on 8-16 bit microprocessor devices.

Theoretical Basis

We consider the change in the properties of the discrete filter $D0\cdot(j\cdot\omega, T0)$ in the frequency domain when the quantization frequency changes N times: fN=f0/N, with constant numerical parameters.

It is known that a fairly good approximation of the transcendental FR $D0\cdot(j\cdot\omega, T0) = D0(e^{j\cdot\omega\cdot To*N})$ in the discrete system in the low frequency range $0 < \omega < 2\cdot f0$ give pseudo-frequency characteristics (Shamrikov, 1985) constructed as a function of the parameter $\lambda = 2\cdot f0\cdot tg\frac{\omega}{2\cdot f0}$, where λ is the absolute pseudo-frequency practically coincides with the real frequency ω, if $\omega \to 0$. We note that the parameter λ in some works on digital signal processing is described by the term "digital frequency". Further we consider the calculation of the parameters of such DF from the condition that the pseudo-frequency characteristics of the reference DF at a given quantization frequency *f0* and pseudo-frequency characteristics of such a filter at a sampling frequency *f0/N* were close within the specified range. Such a criterion can give a good coincidence of the characteristics of a similar filter and a reference filter in the real frequency band $0 < \omega < min(2/T0, 2/T0\cdot N)$. This follows from the fact that the absolute pseudo-frequency λ and real frequency ω within the specified range are close enough. Using the transition to *w*-region, where *w* is the parameter of the bilinear transformation $z = \frac{1+T0\cdot w/2}{1-T0\cdot w/2}$ (Shamrikov, 1985), and equating the characteristics of the reference DF at the calculated frequency *f0* and a similar DF at a changed frequency *fN* we will get the following:

$$D0\left(\frac{1+T0\cdot w/2}{1-\frac{T0}{2}w}, T0\right) = DN\left(\frac{1+T0\cdot\frac{N}{2}w}{1-T0\cdot\frac{N}{2}w}, T0\cdot N\right), \quad (5)$$

whence we write the transfer function of similar a filter in *w* domain in the following form:

$$DN(w) = D0\left(\frac{1+\text{T0}\cdot\frac{\text{w}}{2}}{1-\text{T0}\cdot\frac{\text{w}}{2}}, T0\right) = \frac{A\left(\frac{1-\text{T0}\cdot\text{w}/2}{1+\text{T0}\cdot\text{w}/2}\right)}{B\left(\frac{1-\text{T0}\cdot\text{w}/2}{1+\text{T0}\cdot\text{w}/2}\right)}. \quad (6)$$

Using the transfer function *DN(w)* of a similar filter in *w* domain from (6), and applying the inverse *w*-transformation for the changed frequency *fN*, we pass to *z*-transfer function *DN(z)* of similar a filter, taking into account the changed frequency *f0/N*:

$$DN(z, T0\cdot N) = DN(w, T0\cdot N) \text{ when replacing } w=\frac{2}{T0\cdot N}\cdot\frac{z-1}{z+1}. \quad (7)$$

Let the reference DF be described by *z*-transfer function *D0(z)* of the form (1). Then from expression (6) after transformations we obtain *DN(w)* and, substituting the formula for the inverse *w*-transformation $w = \frac{2\cdot(z-1)}{T0\cdot N\,(z+1)}$, we will get the following:

$$DN(z) = \frac{a_0(z(N+1)+N-1)^n + a_1(z(N+1)+N-1)^{n-1}(z(N-1)+N+1) + \cdots + a_n(z(N-1)+N+1)^n}{(z(N+1)+N-1)^n + b_1(z(N+1)+N-1)^{n-1}(z(N-1)+N+1) + ..b_{n-1}(z(N+1)+N-1)\cdot(z(N-1)+N+1)^{n-1} + b_n(z(N-1)+N+1)^n}, \quad (8)$$

To simplify the expression, we introduce the conversion parameter $R = \frac{N-1}{N+1} = \frac{f0-fN}{f0+fN}$, associated with a change in the quantization frequency (*R=0* at a constant frequency when *fN=f0*).

Taking into account the entered coefficient, dividing the numerator and denominator *DN(z)* on $(N+1)^n \neq 0$ after algebraic transformations, we obtain *z* - the transfer function of similar a filter in the form of the ratio of polynomials *AN*(z^{-1}) and *BN*(z^{-1}):

$$DN(z) = \frac{a_0\cdot(1+R\cdot z^{-1})^n + a_1\cdot(1+R\cdot z^{-1})^{n-1}\cdot(R+z^{-1}) + a_2\cdot(1+R\cdot z^{-1})^{n-2}\cdot(R+z^{-1})^2 \ldots + a_n\cdot(R+z^{-1})^n}{(R+z^{-1})^n + b_1\cdot(1+R\cdot z^{-1})^{n-1}(R+z^{-1}) + ..b_{n-1}\cdot(1+R\cdot z^{-1})\cdot(R+z^{-1})^{n-1} + b_n\cdot(R+z^{-1})^n}, \quad (9)$$

where $z = e^{j\cdot\omega\cdot To*N}$.

Formula (9) defines the way to calculate the coefficients $\{aN_i, bN_j\}$ of *z* - transfer function of similar DF by coefficients $\{a_i, b_j\}$ of the reference filter and the recalculation parameter $R = \frac{N-1}{N+1}$ from the condition of closeness of their characteristics at absolute pseudo-frequencies. For an arbitrary order of DF we obtain from expression (9) rather cumbersome relations for determining the coefficients, which at a high order *n* of the filter will require complex calculations in real time for tuning. In this connection, let us further consider a computational algorithm for obtaining parameters of such a filter: numerator and denominator coefficients (9).

From expression (9) we obtain the algorithm for calculating polynomial coefficients *AN*(z^{-1}) of numerator of the transfer function *DN(z)* of such a filter, using the representation of the polynomial in the form of Horner's method (Aho et al., 2000):

$$AN(z^{-1}) = [..[[a_0]\cdot(1+R\cdot z^{-1}) + a_1\{R+z^{-1}\}]\cdot(1+R\cdot z^{-1}) + a_2\{(R+z^{-1})^2\}]\cdot(1+R\cdot z^{-1}) + a_3\{(R+z^{-1})^3\}]\cdot(1+R\cdot z^{-1}) + ..] + ..]\cdot(1+R\cdot z^{-1}) + a_n\{(R+z^{-1})^n\}, \quad (10)$$

The polynomial of the denominator *BN(z)* is similarly obtained for this purpose in formula (10) it is necessary to replace the coefficients a_i of the numerator polynomial by the coefficients b_i .

We denote the polynomials of degree *j* of z^{-1} in square brackets of expression (10) as $M1(z^{-1}, j)$, and the polynomials in curly brackets as $M2(z^{-1}, j)$. At the same time, the degree *j* of the polynomials *M1* and *M2* varies from 0 to *n*, and the elements of the polynomials are of degree z^{-1} with numerical coefficients.

Initial values of polynomials at *j*=0 are defined as:

$$M2(z^{-1},0)=1, M1(z^{-1},0)=a_0. \quad (11)$$

Subsequent polynomials of degree 1 to *n* from Horner's method are calculated using the recurrence formula:

$$M2(z^{-1},j)=M2(z^{-1},j-1)\cdot(R+z^{-1}), \quad (12)$$

$$M1(z^{-1},j)=M1(z^{-1},j-1)\cdot(1+R\cdot z^{-1})+a_j\cdot M2(z^{-1},j), \quad (13)$$

Calculation of polynomials $M1(z^{-1},j)$ and $M2(z^{-1},j)$ ends at the step *j*=*n*, and the resulting polynomial $M1(z^{-1},n)$ corresponds exactly to the numerator $AN(z^{-1})$ of *z*-transfer function *DN(z)* of similar a filter. Similarly we can calculate the polynomial $BN(z^{-1})$-denominator *DN(z)*, by changing in the algorithm (13) the coefficients of a_i to b_i.

Thus, according to the known polynomials $A(z^{-1})$ and $B(z^{-1})$ of *z*-transfer function *D0(z)* of the reference filter and the values of the actual quantization period *T0*N*, or the polynomials can be calculated using the scaling parameter R $AN(z^{-1})$ and $BN(z^{-1})$, defining the numerator and denominator of *z*-transfer function *DN(z)* of such a filter. Due to the linear relation between the coefficients $\{a_{i,}b_i\}$ of the reference filter and the coefficients $\{aN_{i,}bN_i\}$ of similar a filter, it is convenient to represent this relation in the form of matrix relations:

$$AN=Q(R)\cdot A0, \quad (14)$$

$$BN=Q(R)\cdot B0, \quad (15)$$

where *A0*, *B0* are the *(n+1)*-dimensional parameter vectors $\{a_j\},\{b_j\}$ of DF reference, *AN* and *BN* are the *(n+1)*-dimensional parameter vectors $\{aN_j\},\{bN_j\}$ of a similar DF, *Q(R)* – $(n+1)\times(n+1)$ is the transformation matrix. It is characteristic that the elements of the matrix *Q(R)* transformations have, for a given order *n*, a constant structure and depend only on the parameter *R*. These elements are polynomials of degree *R*, they are not higher than *n*, and the coefficients of these polynomials, integers, are binomial coefficients and depend only on the order *n* of the filter. Thus, for the second-order filter example *(n=2)*, calculating by formula (10), we obtain:

$$AN(z^{-1})=(a_0+R\cdot a_1+R^2\cdot a_2)+(2Ra_0+(R^2+1)\cdot a_1+2R\cdot a_2)\cdot z^{-1}+(R^2a_0+R\cdot a_1+a_2)\cdot z^{-2}, \quad (16)$$

whence the scaling matrix Q(R) is defined by the following expression:

$$Q(R)=\begin{bmatrix}1 & R & R^2\\ 2\cdot R & R^2+1 & 2\cdot R\\ R^2 & R & 1\end{bmatrix}, \quad (17)$$

Methodology

As it was shown above, to ensure the constancy of DF characteristics when the quantization frequency changes, it is necessary in the process of filter operation to carry out the tuning, based on the recalculation of DF parameters. We consider the possible logic of such a tunable filter and the method of its calculation. At the same time, we take into account that the structure of the matrix *Q(R)* depends only on the order of the digital filter, and the numerical parameters, the elements of the matrix, depend on the specific values of *R*.

Due to the fact that the matrix *Q(R)* depends only on the degrees of the parameter *R*, it is convenient to represent this transformation matrix in the form of decomposition:

$$Q(R)=q0+q1\cdot R+q2\cdot R^2+..+qn\cdot R^n, \quad (18)$$

where $qJ(J = 0..n)$ – $(n + 1) \times (n + 1)$ are the matrices, the elements of which are integers, defined by products and sums of binomial coefficients.

Taking into account the decomposition (18), the parameters $\{aN_j\}, \{bN_j\}$ of a similar filter from formula (13) is determined by the following expression:

$$AN = q0 \cdot A0 + q1 \cdot A0 \cdot R + q2 \cdot A0 \cdot R^2 + .. + qn \cdot A0 \cdot R^n, \qquad (19)$$

$$BN = q0 \cdot B0 + q1 \cdot B0 \cdot R + q2 \cdot B0 \cdot R^2 + .. + qn \cdot B0 \cdot R^n, \qquad (20)$$

In the formulas (19) $qJ \cdot A0$ and $qJ \cdot B0 - (n + 1)$ – dimensional vectors, their components are known in advance, because they depend on the parameters $\{a_j\}, \{b_j\}$ of the reference DF and elements of constant matrices $qJ(J = 0..n)$.

Thus, to recalculate the parameters of DF when changing the quantization period it is enough to $2 \cdot (n + 1)$ multiplications of known vectors $qJ \cdot A0$ and $qJ \cdot B0$ $(J = 0..n)$, placed in memory for different degrees of the current recalculation parameter R^J and $2 \cdot n$ additions of the obtained products according to the ratios (18).

Thus, for the above example of a second-order DF we obtain:

$$Q(R) = q0 + q1 \cdot R + q2 \cdot R^2, \qquad (21)$$

$$Q(R) = \begin{bmatrix} 1 & R & R^2 \\ 2 \cdot R & R^2 + 1 & 2 \cdot R \\ R^2 & R & 1 \end{bmatrix}, \qquad (22)$$

$$q0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (23)$$

$$q1 = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}, \qquad (24)$$

$$q2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad (25)$$

$$q0 \cdot A0 = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}, \qquad (26)$$

$$q1 \cdot A0 = \begin{bmatrix} a_1 \\ 2 \cdot a_0 + 2 \cdot a_2 \\ a_1 \end{bmatrix}, \qquad (27)$$

$$q2 \cdot A0 = \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix}. \qquad (28)$$

We use the assumption that the period of incoming data can change significantly during operation, but such changes are episodic and the filter basically works in quasistationary mode with a constant step of sampling *TN=T0*N* and constant parameters. Besides, from practical considerations it is clear that at small enough change of sampling rate | *f0-fN* | < *eps*, *(eps* is a small value) due to the continuity of the parameters does not require.

It is assumed, that parameters *A0* and *B0* of the reference filter are calculated beforehand, and the filter is realized in the form of the program of calculations, similar to (2), for the digital calculator. In this case, the organization of the restructuring is proposed to be carried out at the preliminary stage (labor-intensive operations), and directly in the process (in real time).

Pre-calculation stage:

1. to determine the threshold *eps* value of the frequency change *f0-fN*, above which the filter parameters must be rebuilt;
2. to calculate the constants $(n+1) \times (n+1)$ - dimensional matrices *q0, q1,.., qn*, determining the recalculation matrix, and place them in memory, for which: to calculate the recalculation matrix *Q(R)* from (13) as a function of the parameter *R* using the computational algorithm from (7), to get matrices *qJ* - decomposition components *Q(R)* from (10) and to write them into the memory block – $2 \cdot n^2 \cdot (n+1)$ integers;
3. to write parameters *A0, B0* of the reference filter to RAM – $2 \cdot (n+1)$ real numbers;
4. to calculate $2 \cdot (n+1), (n+1)$ -dimensional matrices $qj \cdot A0$ and $qj \cdot B0$ required for recalculation of parameters by formulas (19,20), place them in the filter memory.

Real-time filter operation:

1. to make episodic measurements of the frequency of incoming information *fN;*
2. if the frequency change threshold is exceeded: |*fN-f0*|> eps, to make a quick recalculation of the filter parameters, to do this:

a. to calculate the current recalculation parameters $R = \frac{(f0-fN)}{(f0+fN)}, R^2, .. R^n$;
b. to calculate the parameters aN_i, bN_i of similar filter for specific values of the parameter R and its degrees, using formulas (19,20), and vectors $qj \cdot A0$ and $qj \cdot B0$, placed in the filter memory in the preliminary stages;
c. to carry out a realignment and to use the calculated parameters aN_i, bN_i in the current operation of DF.

Further, the parameters aN_i, bN_i and the current frequency *fN* can be used as reference parameters a_i, b_i, and *fN* is *f0* and then we go to the step 3.

In the above procedure only items 5 and 6 are performed in real time, i.e. the realignment requires a single execution of uncomplicated and fully defined multiple arithmetic operations of multiplication and addition, while the main time-consuming operations are performed at the calculation stage.

Results

Thus, an algorithm for the operational restructuring of the numerical parameters of the filter based on information about the time intervals of information receipt has been developed. Such a restructuring makes it possible to ensure the constancy of the frequency characteristics of the digital filter with a significant change in the time periods *TN=T0*N* of information receipt and is implemented by simple computational operations.

A technique for implementing the developed recalculation algorithm is proposed, which uses the most cumbersome calculations at the preliminary calculation stage, and during real-time it uses restructuring, performs only single arithmetic calculations with numbers placed in the device's memory.

The developed proposals can be used for an arbitrary order of digital filters, while changing the frequency *fN* of information receipt within a wide range.

The approaches proposed in the work for organizing the restructuring of the parameters of DF were tested on a model example of a fourth-order filter. At the same time, preliminary calculations of the conversion matrices and operational calculations in real time were carried out with a limited calculation accuracy corresponding to 8 binary digits. The change in the quantization period *T0* corresponded to the range of change in the parameter *N=0.3-3*, with a corresponding change in the parameter *R*.

Figure 1 shows the frequency characteristics of the fourth-order low-frequency DF that implements the Butterworth distribution (Oppenheim & Schafer, 2018; Jing et al., 2019). Here, the solid thick line defines

FR of the reference filter at the calculated frequency *f0*, the thin line is the response when the quantization frequency *fN* changes *(N=2.5)*, the dashed line illustrates the response of a similar (tuned) filter. As can be seen from the Figure 1, when changing the step of discreteness of data receipt without restructuring the parameters, the transformation characteristics change significantly. The use of tuning the filter parameters makes it possible to ensure the practical constancy of DF characteristics with a significant change in the quantization frequency, which confirms the validity and prospects of using the developed approaches to ensure the constancy of DF properties when changing the time parameters of the data arrival.

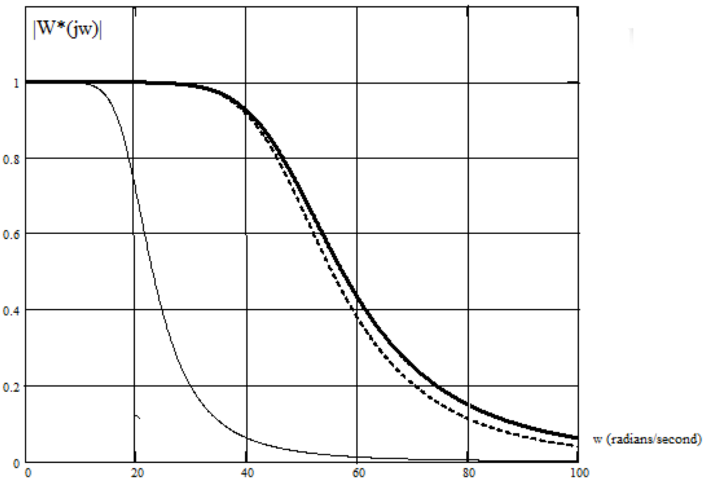

Figure 1. Amplitude-frequency characteristics of DF

A possible specific technical implementation of the described approaches is implemented in a digital device according to patent No. 2631976, described in (Belonogov, 2017; Dimc et al., 2017).

Conclusions

Based on the research presented in the work, the following provisions can be formulated.

- changing the frequency *fN* of DF information arrival relative to the calculated frequency *f0* with its fixed parameters can significantly worsen the required signal processing properties, and when using the results of DF in a closed loop, significantly degrade the system quality indicators.
- technique and a computational procedure have been developed that form the law of rearrangement of the parameters of DF when changing the intervals of information arrival, relative to the calculated values of *T0*. This procedure is used at the stage of calculating the system, and simplified arithmetic algorithms adjust the parameters in real time only when the frequency *fN-f0* changes, exceeding the pre-calculated threshold.
- use of tuning the values of DF parameters to ensure the closeness of the filter characteristics to the characteristics of the reference filter operating at the initial frequency *f0*, allows maintaining the comparative constancy of the filter characteristics with significant changes in the period *T0* of data arrival. At the same time, the algorithms for DF and tuning DF parameters are compact and can be easily implemented on the simplest microprocessor devices.

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