Artículo de investigación

The social and cultural approach to forming geometric concepts among schoolchildren

Социокультурный подход к формированию геометрических понятий у школьников

Enfoque socio-cultural para la formación de conceptos geométricos en escuelas de niños

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Abstract

In line with the social and cultural approach, the problem of forming mathematical concepts among schoolchildren as the system of judgments is considered. Concepts at the verbal and logical level are formed through teaching to prove theorems and solve proof problems. Objective: To provide elements of the method of forming geometric concepts by means of specially organized learning activities of schoolchildren to develop the generalized ability to prove with the access to the value-oriented learning. Methods: Theoretical provisions on the laws of the concept formation process are developed. During the experiment forming stage the methodology to teach geometry to pupils of the 7th grade of secondary education institutions is developed. The basis is the activity to develop skills to prove. Relying on instrument-oriented, subject-oriented and value-oriented types of learning is assumed. Findings: During the quantitative and qualitative evaluation of results, the following independent characteristics have been taken into account: the form of action, the level of generalization, the level of expansion, the level of mastering, and the value relation. The results of analyzing the statistical data hav e confirmed the hypothesis about the significant influence of the following

Аннотация

История вопроса. В русле социокультурного подхода рассматривается проблема формирования математических понятий у школьников как системы суждений. Формирование понятий на вербально-логическом уровне осуществляется через обучение доказательству теорем и решению задач на доказательства. Цель (Objective): Представить элементы технологии формирования геометрических понятий средствами специально организованной учебной деятельности школьников по освоению обобщенного умения доказывать с выходом в ценностно-ориентированное обучение. (Methods): Разработаны теоретические положения о закономерностях процесса формирования понятий. В ходе формирующего этапа эксперимента разработана методика обучения геометрии учащихся 7-х классов среднеобразовательных учреждений. В качестве основы выступала деятельность по освоению умений доказывать. Предполагалась опора на инструментально-ориентированный, предметно-ориентированный и ценностно-ориентированный типы обучения.

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factors on the success of mastering geometric concepts by pupils: the mathematical training (significance level 0,003), the effect of electronic educational environment in combination with the evaluated method (significance level 0,001), the duration of training using the evaluated method (significance level 0,01). Conclusions: The hypothesis is substantiated and proved in practice: the process of forming concepts is regularly ensured by the following psychodidactic conditions: the formation of main components of conceptual psychic structures; the formation of declarative, procedural and evaluative knowledge; the gradual formation of the subjective image of the concept content; the gradual development of the entire psychic structure; the gradual development of the activity component of geometric concepts.

Keywords: geometric concepts; generalized ability to prove; integral units of thinking; real cognitive operations; plane of sign form.

Результаты (Findings): При проведении количественной и качественной оценки результатов учитывались независимые характеристики: форма действия, степень обобщения, мера развернутости, мера освоения и ценностное отношение. Результаты анализа статистических данных подтвердили гипотезу о значимом влиянии следующих факторов на успешность освоения обучающимся геометрических понятий: математическая подготовка (уровень значимости 0,003), влияние электронной образовательной среды в сочетании с апробируемой методикой (уровень значимости 0,001), продолжительность обучения с использованием апробируемой методики (уровень значимости 0,01). Выводы (Conclusions): Обоснована и на практике подтверждена гипотеза: процесс формирования понятий закономерно обеспечивается следующими психодидактическими условиями: формирование основных компонентов понятийных психических структур; формирование декларативных, процедурных и оценочных знаний; пофазовое формирование субъективного образа содержания понятия; поэтапное развитие целостной психической структуры; поэтапное развитие деятельностной компоненты геометрических понятий.

Ключевые слова: геометрические понятия; обобщенное умение доказывать; целостные единицы мышления; реальные познавательные операции; плоскость знаковой формы.

Resumen

En línea con el enfoque social y cultural, se considera el problema de formar conceptos matemáticos entre los escolares como el sistema de juicios. Los conceptos a nivel verbal y lógico se forman a través de la enseñanza para probar teoremas y resolver problemas de prueba. Objetivo: Proporcionar elementos del método de formación de conceptos geométricos por medio de actividades de aprendizaje especialmente organizadas de los escolares para desarrollar la capacidad generalizada de demostrar con el acceso al aprendizaje orientado a valores. Métodos: Se desarrollan disposiciones teóricas sobre las leyes del proceso de formación de conceptos. Durante la fase de formación del experimento, se desarrolla la metodología para enseñar geometría a alumnos de 7º grado de instituciones de educación secundaria. La base es la actividad para desarrollar habilidades para demostrar. Se supone que se depende de los tipos de aprendizaje orientados a instrumentos, orientados a la materia y orientados al valor. Resultados: Durante la evaluación cuantitativa y cualitativa de los resultados, se han tenido en cuenta las siguientes características independientes: la forma de acción, el nivel de generalización, el nivel de expansión, el nivel de dominio y la relación de valor. Los resultados del análisis de los datos estadísticos han confirmado la hipótesis sobre la influencia significativa de los siguientes factores en el éxito del dominio de los conceptos geométricos por parte de los alumnos: la formación matemática (nivel de significación 0,003), el efecto del entorno educativo electrónico en combinación con el método evaluado (nivel de significación 0,001), la duración de la capacitación utilizando el método evaluado (nivel de significación 0,01). Conclusiones: La hipótesis está fundamentada y demostrada en la práctica: el proceso de formación de conceptos se garantiza regularmente mediante las siguientes condiciones psicodidácticas: la formación de los componentes principales de las estructuras psíquicas conceptuales; la formación del conocimiento declarativo, procesal y evaluativo; la formación gradual de la imagen subjetiva del concepto de contenido; el desarrollo gradual de toda la estructura psíquica; El desarrollo gradual del componente de actividad de los conceptos geométricos.

Palabras clave: conceptos geométricos; capacidad generalizada para demostrar; unidades integrales de pensamiento; operaciones cognitivas reales; Plano de forma de signo.

Introduction

At present, in the range of educational innovations, there are notable experiments in mental and action pedagogics. Mental and action educational methods correspond to the social and cultural paradigm of education and have a large innovative potential, expressed in the principle of meta-subject learning.

The new concept of Federal educational standards implies the necessity to provide metasubject educational results. At the same time, the critically-minded part of the pedagogical community complains that standards do not clarify the essence of metasubject educational results, do not reveal the connection between the metasubject content of education and the subject one and a set of means of providing metasubject results in teaching mathematics and other subjects. As a result, in practice, metasubject results are replaced with general education ones, didactic concepts - with psychological ones.

On the other hand, in modern Russian education there are three main ways: 1) the form of subject knowledge and skills remained from Soviet pedagogics, 2) the form of action, and mental and action education, and 3) the information form of the Unified State Exam. The form of action, and mental and action education meets the provisions of the social and cultural approach, which considers value as the leading category. The psychophysiological basis of the social and cultural approach is the theory of functional systems, created by P.K. Anokhin in the framework of the activity approach. The socialization in line with this approach acts as the process of mastering structural components of the activity by the individual, as well as its forms such as perception, academic, mental activity in the field of mathematics, a special place in which belongs to concepts.

All of the above said actualizes the problem of forming geometric concepts among schoolchildren in the context of the social and cultural approach as the system of judgments, which is possible only when learning to prove theorems and solve proof problems.

The purpose of this article is to present elements of the method of forming geometric concepts by means of specially organized educational activities of schoolchildren to develop the generalized ability to prove with the access to value-oriented learning.

Literature Review

Vygotsky (2016) acted as the supporter of the process of forming conceptual thinking in the course of studying sciences and mastering scientific concepts. These theses were confirmed by Vekker (1976) as part of the experimental study of students' and researchers' thinking.

Piaget (1966) speaks about two types of main tools of perception: "... on the one hand, descriptors characterizing states or transformations, on the other hand, operators or combinators, allowing to reproduce transformations and use them, taking into account their initial and final state. "

Talyzina (1984) highlighted stages of forming scientific concepts. Kholodnaya (2002) was engaged in developing the technique to form the conceptual thinking in the context of forming the cognitive component of the mental experience of the individual as the indicator of the level of formation of the convergent abilities of the individual.

The process of mastering mathematical concepts by schoolchildren was studied in the works of

Vering (1989), Vilenkin, Abaydulin, Tavartkiladze (1984). Vering (1989) believed that the mental activity does not develop effectively if one tries to influence it only through the study of academic subjects, including mathematics. Vilenkin et al. (1984), studied issues of the personality development in teaching mathematics.
A special attention is traditionally paid to the process of mastering concepts when analyzing the study of the school geometry course (Voistinova, 1984).

Fahlberg-Stojanovska and Stojanovski (2009) have found out that the use of GeoGebra positively affects the motivation of trainees and improves the knowledge level in general.

Aydin and Monaghan (2011) described in their researches that with the help of GeoGebra, trainees had the opportunity to see mathematics in everyday life using such a dynamic geometry system in the classroom.
In their studies, Thambi and Eu (2013) investigated the academic progress of pupils with the help of fractions using GeoGebra software.

In their study, Zengin, Furkan, and Kutluca (2012) found out that the software for dynamic mathematics, such as GeoGebra, had a significant effect on teaching elements of trigonometry.

Hall and Chamblee (2012) concluded that the main mechanism to improve teaching and learning mathematics in the middle school was the implementation of GeoGebra software.

The results of Zakaria and Lee (2012) showed that teachers of mathematics in secondary schools had a positive attitude towards using GeoGebra.

Based on the results obtained during the experiment, Takachi, Stankov and Milanovic (2015) determined that GeoGebra helped all pupils to study mathematical concepts.

Arbain and Shukor (2015) in their researches state that studying and teaching mathematics should be focused not only on purely theoretical, but also on various approaches to learning.

Ochkov and Bogomolova (2015) in their research came to the conclusion that computers could take on the routine mathematical work, allowing the teacher and pupils to create something more exciting in the process of learning.

Methods

During the experiment forming stage the methodology to teach geometry to pupils of the 7th grade of secondary education institutions was developed. The basis for this was the activity to develop skills to prove geometric propositions. Actions that constitute the content of the ability to prove were defined in its structure. The content of the activity structure was determined by the classification of the geometric concepts mastered according to the level of the thinking process operations (real, formal and integral).

We should note that the modern metasubject approach to teaching mathematics postulates: while proving propositions, the pupil simultaneously performs the reflexive analysis of the component composition of the ability to prove, which ensures obtaining a metasubject result — the pupil distinguishes between a proof method and a generalized method of mental activity by proving a theorem.

Previously, such categories as concept, property, attribute, etc. were practiced. Next, the work was carried out to form such actions as part of a generalized way of proving propositions, for example, deriving consequences from what is given in the statement, or bringing the geometric phenomena specified in the statement to systems of attributes of required concepts, etc. We should note that traditionally in researches devoted to the problem of teaching to prove, the actions that constitute the content of the ability to prove do not act as a special subject of mastering. In the traditional school methodology, no attention is paid to what method is used to prove. The only exception is the proof by contradiction.

In the framework of the experimental action methodology components of the content of the ability to prove, acted as a special subject of mastering. So, in the composition of the generalized ability to prove, we selected components (methods of activity), and the criterion to develop the skill was not only mastering these methods of activity, but also the metasubject skill to analyze the ability to prove - to reveal its content, to single out its components. The main components of the ability to prove are the skills listed below, which should be considered as mental process techniques – integral operations. Let us reveal the content of the main components of the ability to prove.

1. The contensive analysis of what is given and what is required to be proved. Here we note real cognitive operations of the thinking process,

starting directly with a geometric object, representing actions with the object, and we select a certain content in the object, making the “transition from objects to drawing models”.

2. The deductive derivation of the consequences of what is given in the statement. Going from the drawing to formal relations, it is necessary to pay attention to the abstract nature of geometric figures. Next, there are operations in the plane of the sign form: relying on formal mental operations, starting with ready-made knowledge, verbal expressions, fixing the "link of knowledge", which allows to move from one object property to another.

3. The ability to reason using the synthetic method.

4. The ability to reason using the bottom-up method (or Pappus analysis, the perfect analysis).

5. The ability to perform the indirect (top-down) analysis (the method by contradiction or Euclid’s analysis, nonperfect analysis).

6. The action of bringing the phenomena specified in the statement to the system of concept attributes (the analytical and synthetic reasoning).

7. The selection of conjunctive and disjunctive forms of concepts.

8. The selection of direct and inverse propositions, etc.

Specially organized educational activities and a special system of tasks are considered to be a didactic condition of the effective formation of such skills. The main goal orientation is mastering the reasoning method by the pupil, the self-regulation of his mental activity in the proving process. Pupils used a special rule that revealed the content and sequence of actions. At the same time, the pupil should have a clear idea that in each item of the said rule it is recommended to perform actions presenting quite complex skills that constitute the content of the ability to prove: the contensive analysis of what is given and what is required to be proved; the deductive derivation of the consequences of what is given in the statement; the ability to bring the phenomena specified in the statement to the system of attributes of required concepts (to prove synthetically the skills constituting the content); the ability to reason using the bottom-up method; the ability to perform the indirect analysis (the method by contradiction); etc. Mastering the concept is the result of mastering these actions. Let us consider how it looks using the example of a theorem from the textbook of A.V. Pogorelov.

Theorem 6.2. The parallelogram diagonals intersect, and the intersection point divides them in half.

Proof:

The contensive analysis of what is given and what is required to be proved. Here we speak about real cognitive operations of the thinking process, starting with geometrical objects (in particular, a parallelogram). Let ABCD be a parallelogram (Figure 1).

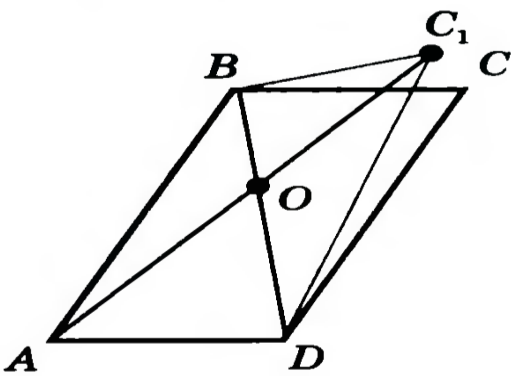

Figure 1. The ABCD parallelogram’s analysis

Let us draw its diagonal BD. Let O be its midpoint, and on the extension of the segment AO we intercept the segment OC1 equal to AO. Let us move from the drawing to formal relations: we draw attention to the abstract nature of the geometric object “parallelogram”, we say that it exists only in our imagination, in our thoughts.

The ability to perform the bottom-up analysis.

In order to prove the theorem, it is enough to prove that the points C1 and C coincide. To do this, it is enough to prove that the straight-line BC coincides with the straight line BC1, and the straight line DC1 coincides with the straight-line DC.

The ability to perform the indirect analysis.

Let us assume the contrary, namely, that these straight lines do not coincide. According to theorem 6.1, the quadrilateral ABC1D is a parallelogram. Therefore, the straight line BC1 is parallel to AD. But through the point B you can draw only one straight line parallel to AD. We have got a contradiction. Therefore, the assumption was wrong. Hence, the straight line BC1 coincides with the straight line BC.

In the same way, it is proved that the straight line DC1 coincides with the straight line DC.

Hence, the point C1 coincides with the point C. The parallelogram ABCD coincides with the parallelogram ABC1D. Therefore, its diagonals intersect, and the intersection point divides them in half. The theorem is proved.

It should be noted that the priority in our methodology was the task of accessing the value-oriented learning. The thing is that any object under consideration is characterized in relation to a person, his norms, ideals and goals. In this sense, learning is considered to be a gradual accumulation of the value knowledge system, which is expressed in the form of value judgments. The sensor-emotional way of information coding is assumed, which is formed thanks to questions that lead pupils to emotional evaluations of the subject material, creating virtual situations in which the pupil can use metaphors, imagination and fantasy.

As one of the means, we used the reference to one of types of cultural and historical analogues - multi-scientific methods of solving the same problems. So, for example, we turned to one of the fundamental educational objects - the parallel axiom, the etymology of which was connected with the problem of the fifth Euclidean postulate. Pupils were offered two analogues of its interpretation - Euclidean geometry and absolute geometry. The preliminary selection of the material provided the opportunity for schoolchildren to see the role of the problem to prove the fifth Euclidean postulate in universal culture, world-view aspects of this problem, to realize the genesis of scientific views, discoveries, to get acquainted with the work of mathematicians (K. Gauss, J. Bolyai, N. I. Lobachevsky), to evaluate the dramaturgy of their professional fate, and etc. All this contributed to leading pupils to emotional evaluations of the subject material using such words as “important,” “curious,” “interesting” and etc., creating the “educational tension”, which required pupils to make a choice from the analogues offered by the teacher according to their world-view coordinates.

Results

From the perspective of formal logic, a mathematical concept is a complex system of interconnected, logically ordered judgments. The content of the concept is revealed in the system of judgments, that is why it is usually associated with acts of reasoning. It is not so easy to see and understand, because we are accustomed only to distinguish the “content” and “form” of the concept, and it is not reasonable to replace the phenomenon of the “mathematical concept” with the mathematical object (this principle of understanding has developed in minds of teachers and methodologists, and it is very difficult to overcome). Within the framework of this article, the attempt is made to discern the essence of forming the mathematical concept as the system of judgments, which is possible only when learning to prove theorems and solving proof problems.

Distinguishing the "mathematical concept" and "mathematical object", we proceed from the fact that theoretical ideal objects constitute the subject of geometry. By geometric objects in a broad sense, we mean topological manifolds, as well as binary relations (equalities, parallelism, etc.), ternary relations (the composition of motions, scalar product of vectors, etc.).

Following psychologists, we refer means of the mental reproduction of geometric objects to the plane of the geometric knowledge content, and descriptions of geometric objects - to the plane of the sign form. Then geometric concepts as means of the mental reproduction of objects unite these planes. With this approach to understanding the geometric concept, the specific character of conceptual thinking is determined as follows: it establishes itself in image structures, with logical operations being the development mechanism (Ustylovskaya, 2008).

Following Shchedrovitsky (2003) we consider integral operations as units of the integral thinking process, including real operations and formal ones, ensuring the unity of the content plane and the sign form plane. Real operations are presented as actions with the object and determine a certain content in the object. Formal operations are not associated with seeing a new content in the object, playing the role of cognitive ones, not being such (Shchedrovitsky, 2003).

If we correlate the selected integral units of thinking with the presentation of the geometric proof, then each operation "sews" four types of actions: 1) "movements" according to the drawing (real operations; the content plane); 2) from the drawing to formal relations; 3) in the plane of the sign form (formal operations, the sign form plane); 4) the interpretation of the obtained result in the drawing.

When making a quantitative and qualitative evaluation of results, we assumed the following characteristics: the form of action, the level of generalization, the level of expansion, the level of mastering, and the value relation.

Taking into account the form of action, we should note that we considered each component of the generalized ability to prove in two aspects: real cognitive operations, formal operations. Units of geometric thinking are integral operations.

Considering the level of generalization, we should note the generalized nature of the mental ability to prove.
In order for a skill to be called expanded, forming the skill to prove, trainees should perform all operations constituting the content of this action.

The level of mastering is based on quick performing of actions and implies a certain level of automation. The mental ability to successfully perform the geometric proof, in contrast to the skill, is not associated with the stage of automation.

The value relation is the acceptance of a logical proof as a value by schoolchildren. Understanding the methodological meaning of the proof as the reflection of cause-effect relations in a deductive form, the ability to evaluate the role of the mathematical proof in human culture, and etc., are assumed.

Discussion

Pre-profile classes (of the physical and mathematical profile) were determined as participants of the experiment: SBEI GES No. 2070 of Moscow KSD (Kommunarskoye School Department) – grades 8 "a" and 9 "a", each having 30 pupils; SBEI GES No. 2070 of Moscow BSD (Buninskoye School Department) – grades 8 "v" and 9 "v", the total number of pupils is 28 and 29, respectively. The distance learning within the framework of the club "Constructive geometry on the Euclidean Plane" was an additional optional form of the subject preparation and a significant mechanism supporting full time lessons on the subject. We focused on the three-stage presentation of the material to be learnt: the stages of understanding, mastering and application. We tested the hypothesis of the isolated and cumulative effect of the following factors on the success of mastering geometric concepts by trainees: the level of the pupil mathematical training; the level of mastering the system of dynamic geometry (GeoGebra); the duration of the training using the evaluated methodology, which assumed a direct correlation of mastering the generalized way to perform geometry constructions and proof in the teaching-learning situation and the process of mastering geometric concepts directly related to it.

The degree of mastering geometric concepts by schoolchildren was diagnosed at two levels – a personal and subject one. The value relation was considered to be a parameter of the personal level. Table 2 shows the results of the diagnostics of the development of this indicator among EG 1 and EG 2.

Table 2. The dynamics of developing the value relation to geometry among pupils of EG 1 and EG 2 (people).

Level	EG 1		EG 2	
	Before EER	After EER	Before EER	After EER
Low	22	13	21	4

Average	22	29	25	23
High	6	9	6	25

The value relation changed in EG 2 at a greater rate than in EG 1. During empirical observations it was found out that the interest of pupils in learning in the electronic educational environment (EEE) without the methodological support of this process after the first increase reduced significantly, while the interest of pupils in EG 2 in learning in EEE increased constantly.

During the diagnostics of the subject level of forming the ability to prove among schoolchildren, the following levels were defined: the reproductive level, which records the development of pupils' ability to prove at the level of understanding; the productive level, suggesting the development of their skills to prove as the application of theoretical knowledge in the process of solving standardized problems and transferring these skills into the skill automated state; the productive and creative level corresponding to the ability to apply the learned subject skills to prove in new non-standard conditions of solving problems requiring additional knowledge not included in the program thesaurus.

The results of the analysis of the statistical data confirmed the hypothesis about the significant influence of the level of the mathematical training (significance level 0,003), the level of the influence of EEE in combination with the evaluated method (significance level 0,001), the duration of the training using the evaluated method (significance level 0,01).

Conclusion

Within the framework of this article, the characteristics of the author's method of forming geometric concepts by means of specially organized educational activities of schoolchildren to develop the generalized ability to prove with the access to value-oriented learning are provided.

The didactic condition for the effective formation of such skills was a specially organized learning activity and a special system of tasks.

The hypothesis confirmed in practice is substantiated: the effectiveness of the process of forming concepts among schoolchildren is ensured by psychodidactic regularities:

- the formation of main components of conceptual psychic structures (means of information coding, semantic structures, concept attributes);

– the formation of declarative, procedural and evaluative knowledge;

the gradual formation of the subjective image of the concept content: motivation, categorization, transfer, folding.

the gradual development of the entire psychic structure “image - representation - preconcept - concept - system of concepts”;

the gradual development of the activity component of geometric concepts: subject actions, real cognitive operations, formal operations, integral operations.

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