Design and implementation of adaptive technology for teaching mathematics to school children based on integrated diagnostic approach to subject preparation and competence development

Abstract

The development of school teaching systems to enable effective adaptive communication of information requires specific pedagogical solutions to several important theoretical and methodological problems. These include 1) the discernment of basic characteristics needed to diagnose and improve the quality of subject preparation for schoolchildren, 2) clarifying the role of the teacher in the structure of adaptive learning process, and 3) finding ways to integrate adaptive content into the framework of courses on mathematics. The purpose of our study was to determine theoretical and methodological foundations of teaching mathematics to schoolchildren taking into account their level of ability with the consequent development of appropriate adaptive content.

The basic characteristics that underpin the concept of adaptive learning which contributes to both individual profiles of student ability to learn and subsequent success outcomes are: proficiency, motivation to learn and level of mathematical knowledge. The evaluation of individual profile structure of schoolchildren

Artículo de investigación

Разработка и внедрение адаптивной технологии обучения математике школьников на основе комплексного диагностического подхода к предметной подготовке и развитию компетентности

Recibido: 8 de octubre del 2019 Aceptado: 26 de noviembre del 2019

Published in: Amazonia Investigacíon

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determines the choice of methodologies for presentation of adaptive content in ways allowing development and motivation. The system of educational process management developed in this way includes both content-methodical and procedural-technological components. This makes it possible to automatically evaluate the level of each students' mathematical training (knowledge, motivation, development) and to ensure continuous improvement.

This system can also be used by secondary teachers of mathematics as a part of extracurricular activities, or as a distance learning support. In addition, the recommendations for structuring multi-level problem material can be used by mathematics teachers to self-construct adaptive sets tasks at various stages of teaching mathematics. As a result, students have the opportunity to improve their own profile of learning success, particularly by solving a chain of tasks.

**Key Words:** Adaptive learning technology, students' motivation, student success profile, structure of mathematical tasks, teaching mathematics.

**Introduction**

It is well known that mathematics has for centuries been considered to be a vital contributor to the development of a child’s intellect and personality. Ensuring such development in the conditions of mass education requires the organization of learning to respond to the individual characteristics of the child, allowing him or her to design a personalized ways of realizing potential through educational process (Rodionov, et al 2010). One of the main ways to implement such an approach in teaching mathematics is to manage the cognitive activity of students with the help of a specially designed set of mathematical tasks at different levels of complexity, which correspond to individual capabilities. When these tasks are carried out studies have shown improved logical processing of information and motivation, not only in mathematics but also in general educational and interdisciplinary competencies (Abramova, et al 2018).

To best achieve this, it is advisable to organize the presentation of educational material in the format of high-tech computer adaptive learning, enabling automatic adaptation of the presentation to respond to students’ individual characteristics (Rodionov, et al 2006). The most well-known testing systems which operate in automatic mode and are available in the global network are DreamBox, CarnegieLearning, KnowRe, Querium, LearnSmart Advantage, TestOfficePro, Knewton, Siberia-Softprograms and LearningManagementSystem. The main value of such systems is to provide the possibility of changing the order of questions depending on the answers of the person being tested. At the same time, it is possible to automatically select tasks...
directly from an algorithmic bank or portfolio which can adaptively produce new assignments based on those which have been completed. These assignments can be individualized. This approach, as many educators - scientists indicate, increases the likelihood that a student will receive the necessary educational content at the right time and achieve the goals he or she has been set. Successful completion is continuously built upon (Rodionov, et al 2007). Equally, if a student finds it difficult to cope with a specific set of questions, the system can provide a modified approach to enable the difficulty to be overcome (Rodionov, et al 2017). Various aspects of adaptive learning and categories associated with it ("Adaptive Internet Technology", "Adaptive Tests") are quite widely reflected in the scientific and pedagogical literature (Rodionov, et al 2008). In particular, the didactic and developmental prerequisites for using such training in real school practice are reflected in a number of works by educators and psychologists (Akimova, et al 2016b). There are also studies directly considering the possibilities of adaptive technologies in the wider educational process (Rodionov, et al 2016b). Among them there are studies which describe limited use of adaptive testing tools to organize control of students' knowledge including the development of an online adaptive testing system to monitor the knowledge of university students (Bulaeva, et al 2018). More promising is an approach that considers adaptive technologies as the basis for organizing the entire learning process, and not just adaptive testing (Bartkiv, et al 2018). Some argue that such an adaptive approach should in the future serve not only to manage the dissemination of knowledge, but also to design individual learning trajectories (Vaganova, et al 2019a). Such an approach can find its application in diverse fields of knowledge, both in the sciences and in the humanities. In particular, S.A. Crossley and D. McNamara are considering the possibility of adaptive learning in literary reading (Vaganova, et al 2019b). S. Bretschneider and V. Mahajan are considering the possibility of adaptive learning in sociology (Vaganova, et al 2019c). In their study, it is proposed to build, on the basis of adaptive testing, a holistic pedagogical technology, which would be an "innovative software product that organically combines: an orientation to interdisciplinary training, a reflection of the specifics of professional pedagogical activity and monitoring the quality of education, the possibility of students' self-control" (Vaganova, et al 2019d).

An analysis of theoretical, methodological and technological solutions to various subject matters shows that most cover only a limited functional range (Vaganova, et al 2019e). In particular, none of the software products presently on the market provides sufficiently effective specialized tools to explicitly identify and consider the cognitive and personal characteristics of children in the field of mathematical education, especially with regard to adaptation for schools (Rodionov, et al 2017b). The challenge of of producing an objective choice of latent parameters of learning, the appropriate structure of cognitive activity and the identification of personal characteristics are all matters which are not fully resolved (Rodionov, et al 2017b). The development of diagnostic and formative procedures must address each of these areas (Vaganova, 2019f).

In other words, whilst the general ideology of adaptive testing and its technical implementation can be considered to be well developed, the pedagogical soundness of the content presented in most systems requires additional research (Rodionov, et al 2017b). There are presently no effective available software solutions in this area, which would allow us to naturally individualize work with schoolchildren of various levels of learning success in the field of mathematics, and thus ensure their effective intellectual and motivational development (Rodionov, et al 2018a). All of the above emphasizes and underscores the relevance of the research topic, the purpose of which was to develop a system of adaptive teaching mathematics to school students, taking into account their individual typological features and the available technology for its implementation in the real world of school practice (Rodionov, et al 2014).

The following tasks were examined in our study:

1) Developing the concept of adaptive learning in mathematics, taking into account pupil levels of subject preparation, competence and potential.
2) Designing an educational and methodological framework which ensures the functioning of the adaptive learning system in accordance with the stated objectives.
3) Developing an adaptive technology for teaching mathematics - an instrumental system that reflects and enables the implementation and realized of any given content.
4) Testing and implementation of technology in the real practice of teaching mathematics to schools.
5) Analysing the results of testing and further developing measures to improve the components of the system.

In the next section of the article we will reveal the features involved in meeting these challenges.

**Theoretical framework**

In developing the concept of an adaptive learning system which takes into account the individual typological characteristics of the student, we investigated the structure of the learning success of each student (Vaganova, 2018). The following triad of characteristics was chosen as the basic structural unit of such success, which we used to establish a system of diagnostics and improvement: the student's learning (S), which characterizes the subject-specific competences he or she has mastered; learning motivation (C), which determines the cognitive activity of the student, and the level of mathematical development (T), which is manifested in solving problems of a research and exploratory nature based on the productive interaction of various components of thinking. The manifestation of the combination of these characteristics by a student at any given time, corresponding to a particular level of learning success, predetermines the immediate prospects for individual intellectual and personal growth (Garnevska, 2018). A low learning success rate correlates with exclusively situational activity, initiated mainly by external motives (Guseva, 2013). Various components of thinking are often fragmented, resulting in repetitive and not developmental educational and cognitive activity. At the most limited level, the student’s individual experience is not coherent and integrated, but, rather, reflects a limited set of techniques for solving typical problems. At the middle level, students perform active learning activities, guided by a variety of external and internal motives. Simple interactions between the conceptual and figurative components of thinking can configure an orientation towards finding a common mode of action, often on an intuitive basis. At the same time, within the framework of a specially organized activity, an effective mechanism of goal formation and meaningful formation begins to manifest itself. Finally, a high level of success requires active independent actions of the student, solely initiated by internal motivation. It is characterized by a theoretical application of knowledge, which allows the effective mastering of mathematical activities at a large degree of abstraction. Here, pupils set learning goals for themselves, often looking for non-standard ways to achieve them. As a result, they develop a systematic formation of knowledge, successfully demonstrating complex forms of interaction between various components of thinking. Based on the simplest combinatorial analysis of possible combinations in the levels of the learning success components development, we can distinguish 15 possible variations of the object <CTS>, a graphic representation of which illustrates two main stages in improving the quality of schoolchildren's learning. Figure 1 shows the possible transitions from one state to another, based on a discrete organization of educational activities. Successfully addressing and facilitating these transitions requires a careful analysis of achievement, aptitude and areas requiring improvement with regard to each indicator.
The selected components of learning success are latent variables that form the basis for the development of the proposed technology for adaptive mathematics education. This in turn provides an adequate definition of the type and level of student success recorded in the relevant success profile, and the creation of favourable conditions for the improvement of weaknesses. The features of adaptive learning systems are, firstly, an analysis and diagnosis of students’ present mathematical competence; secondly, a careful cross referencing with the prevailing curriculum, and, thirdly, a fine tuned supplementation of coursework with research-related material. The structure can be adapted and customized to make general improvements in various knowledge components. Accordingly, the previous scheme (Fig. 1) can be modified as follows (Fig 2) where the lines mean transitions from one state to another.

Fig. 1. Variations of the <CTS>

Fig. 2. The modified of the<CTS>
Methodology

An integral assessment of the type and degree of the student success in the field of mathematics requires an initial diagnosis of the development of objective thinking in each student, the nature of his or her motivation (external, internal, mixed) and level of competence (components: C, T or S) based on special diagnostic tasks. Some of the components of the triad of signs that characterize the quality of student training may be weaker or stronger than others. The result of the current structure analysis to estimate learning success of students determines the nature of methods and forms choice and the subsequent meaningful processing of digestible subject content. This is ensured by a purposeful content and structural transformation of any given task, leading to the construction of a new related task. The significance of such work, as shown in the studies of MA Rodionova, Z. Dedovets (Dedovets, 2015), I.V. Akimova, N.N. Temple, E.V. Guseva and other researchers stems from the fact that if a teacher properly creates an initial learning target, then it does not ‘die’ when the solution of the initial task is completed, but is transformed and transferred to a new subject content and its attendant solution (Chirva, 2018). A developmental system of tasks is formed, consistently improving the overall framework of the mathematical section under study. Each such system includes three parts.

In the first (obligatory) part, it is proposed to solve a typical type of problem, which checks the formation of specific subject knowledge and skills among schoolchildren.

The second (additional) part includes questions where the students have no clear guidelines for the solution. The purpose of raising these questions is to give students some “hint” about the possibility of developing the initial problem situation. Pupils’ attempts to find answers in the second part of the task indicates the motivational potential of the material and how they demonstrate this motivation.

The third part, in essence, is a complicated task, relatively independently compiled by a student in a manual or automated format. The peculiarity of these tasks, examples of which are presented in the studies of MA. Rodionova, N.N. Temple, J. Dedovets, S.L. Velmisova, T.A. Chernetsky consists in providing an opportunity for the student to update their ability to independently set goals (in particular, when drawing up new tasks on the basis of the problem solved in the first part); to find the most common way to act in the proposed task situation; to achieve an effective “translation” of the information laid down in the original problem into an alternative mathematical language, and to develop a fully-fledged reflection of the meaningful interrelationships which are revealed.

Results and discussion

The system of adaptive technology of teaching mathematics we have developed is intended for students of forms 2-3 and includes 16 modules, which correspond to the topics of the course of mathematics reflected in the current curriculum. As part of this project, students who perform a chain of tasks carry out a self-diagnosis of their current level of success and then realize their individual trajectory while mastering the corresponding mathematical content in an automatic mode.

The proposed content can fulfill the role of supporting other content-methodical tools for all students and at all stages of the educational process: diagnostic, formative and reflexive developed in this way can be embedded within, and thus adapt, basic coursework mathematics. This is shown in more detail below.
Diagnostic stage.

At this stage, a student is offered an introductory test consisting of tasks of different levels of difficulty. Each task is characterized by certain indicators S, C and T (one of three parameters) at different levels of difficulty. The total number of tasks is determined by the heuristic potential of the considered task material. The nomenclature of the procedures that make up this potential is determined by the characteristics of all the basic aspects of mathematics thinking to the extent accessible within a secondary school. These procedures, corresponding to the very nature of creative mathematical thinking, are reflected (both individually and in various combinations with each other) in the subject search activity of the student.
Each diagnostic task checks individual experience of the student (S), his or her learning motivation (C) and the level of mathematical development (T). The individual experience of the student, including specific subject knowledge and skills necessary for mastering, is checked using a typical task. Motivation is measured by the production of a new task situation, and finally development is checked based on looking at the student’s creative abilities when producing a solution of a non-standard nature, but nevertheless thematically related to the previous task. We give examples of a comprehensive diagnostic task.

**Task Type S**

Solve the equations and fill the table 1 by writing the roots of the equation in the right column. For multiple roots, write them in ascending order. Write down all fractions as decimal.

<table>
<thead>
<tr>
<th>Equations</th>
<th>Equation roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x - 7 = 3 + 2x</td>
<td>x = 6, 0.91</td>
</tr>
<tr>
<td>4(3x - 2) - 10 = 2(x + 5)</td>
<td>x = 3.2</td>
</tr>
<tr>
<td>( \frac{2}{3} \cdot \frac{1}{9} = 0.6 \cdot x )</td>
<td>x = 0.22</td>
</tr>
<tr>
<td>( \frac{5}{6} \cdot \frac{3}{4} + 1 = \frac{2}{3} - \frac{1}{6} )</td>
<td>x = 5</td>
</tr>
</tbody>
</table>

**Fig. 4. Scheme of diagnostic testing**

**Table 1. Solution of the task type S**
Hint:

To solve equations, use the properties of equations known to you, related to transferring terms from one part to another or dividing an equation by the same number, not equal to 0. To solve the third equation, apply the proportion property. To solve the fourth equation, multiply both sides by the smallest common denominator of all fractions contained in it. To solve the fifth equation, remember how the modulus of the number is and think about what numbers you could substitute for x so that the equation turns into a true numerical equality.

Task Type C

Fill missing data for solving the equation $Ax = B$, where $A$ and $B$ are some numbers using the statements below in the scheme (Fig. 5).

The transfer statements the equation $Ax = B$ (Fig.6)

- $Ax = B$
- $A = 0$
- $B = 0$
- $0x = 0$
- $x = \frac{B}{A}$
- $0x = B$
- unique number
- no solution
- $x$ - any number

Fig.5. Illustration for example 1

Fig.6. Illustration of the transfer statements
Hint: The equation in the form \(0x = 4\) has no solutions; a solution for the equation in the form \(0x = 0\) is any number.

Task Type T

At what values of the number \(a\) does the equation \(a(a + 1)|x| = a\) have two roots?

a) has two roots;
b) has no roots;
c) has an infinite number of roots?

Answer:

a) The equation has two roots, when \(a \neq 0, a \neq -1\).
b) The equation has no roots, with \(a = -1\);
c) The equation has an infinite number of roots, for \(a = 0\).

Hint: Think for what values of \(a\) the equation takes the form \(0|x| = 0\) or \(0|x| = a\).

Methodical comment: The first task is aimed at checking the ability of students to solve equations using the equivalent transformations of equations they know, the property of proportion and the definition of the module. If gaps are found in the required knowledge, students will be offered exercises to fill in the missing skills. The proposed tasks are accompanied by detailed instructions and theoretical information, which a student can refer to if necessary. When completing the second task, students are involved in the construction of partially acquired new knowledge in a theoretical sense (the construction of a solution scheme for the equation in general) (Denysenko, 2018). Here it is required not only to solve the equation, but to understand a general approach and express it in the form of a block diagram. In a task of type T, it is necessary to apply a complex ability to solve equations with a module and with a parameter in a way which demonstrates the ability to reason (Ihnatenko, 2018).

Based on the diagnostic results, a report is generated in the educational process management system that displays for the teacher, students and parents an individual profile of the formation of diagnosed personality components of a particular student (Fig. 7). The total number of points can range from 0 to 8. This profile allows the teacher to evaluate prospects and outline directions for further work for each student, consisting of the selection of tasks corresponding to the current profile of success of the type and level (Koshechko, 2018).

![Fig.7. Levels of formation of the diagnosed components of the identity of individual each student](image)

Formation stage.

After constructing a profile of achievement and for improvement, the student is presented with an individualised programme as follows: (“1C: Mathematical Constructor” is used for constricting and analyzing this material):

- theoretical materials (training text, algorithms, tables, diagrams, graphs, formulas, etc.);
- practical tasks of a basic and advanced level for mastering the studied subject.
content at the level provided for by the programme (their number depends on the structure of the corresponding algorithmic prescription); individual tasks to improve areas of weakness (Klinkov, et al 2018).

Each of these individual tasks relates to one or other of the success components and similarly corresponds to a basic or increased level of difficulty. For example, if the success profile is characterized as S1C2T1 (does not have enough basic material; cannot solve problems of both basic and advanced levels, but has a pronounced search motivation and a desire to experiment with numbers and figures) (Makhomet, 2018), then the educational process management system offers the student tasks of the following types: for T1 all T tasks are offered; for S1 offers all common tasks. We give examples of such tasks (Ivanova, 2019).

Theme "Central and axial symmetry"

Typical tasks of type S (criterion S is checked - the student's learning)

Problem 1.

The segment is given by the coordinates of its ends: A (2; 4), B (4; 2) (Fig. 8).

(a) Draw:

1) the segment $A_1B_1$ symmetric to the segment $AB$ relative to the axis $Oy$;
2) the segment $A_2B_2$ symmetric to the segment $AB$ relative to the axis $Ox$;
3) the segment $B_3A_3$ symmetrical to the segment $B_1A_1$ with respect to the axis $Ox$.

(b) Indicate in the drawing, the point and the required segments:
1) the center of symmetry of the polygon $ABB_2A_2A_3B_3B_1A_1$;
2) a segment centrally symmetric to the segment $AB$;

Answer:
1) point $O$;
2) segment $A_1B_3$;

Hint: Points $A$ and $A_1$ are called symmetric according to point $O$ if point $O$ is the midpoint of segment $AA_1$.

![Fig. 8. Illustration of problem 1(a)](image-url)
Points A and A1 are called symmetric with respect to the line l if the line l is perpendicular to the segment AA1 and divides it in half.

Problem 2.
Determine the shapes:
1. possessing central symmetry;
2. possessing axial symmetry;
3. having at least two axes of symmetry
To answer the question, drag each of figure to the correct cell.

Fig. 9. Illustration of example (b)

Fig. 10. Illustration of problem 2, exp.2
Table 2. Solution of Problem 2

<table>
<thead>
<tr>
<th>The figures having the centre of symmetry</th>
<th>The figures having a symmetry axis</th>
<th>The figures having not fewer than two axes of symmetry</th>
</tr>
</thead>
</table>

**Hint:** A figure is symmetric according to point O if for each point of the figure a point symmetrical to it according to point O also belongs to the figure. A figure is symmetric according to the line l, if for each point of the figure a point symmetrical to it according to the line l also belongs to the figure.

**Conclusions**

The developed adaptive automated learning system was tested from 2016 to 2019 in a number of schools in Moscow, Penza, Khabarovsk and the Moscow Region (240 students of forms 2-3 attended by 8 teachers of mathematics with different experience). Its main goal was to assess the applicability of the developed adaptive content for students in that age range within the area of study. The system itself was deployed by internet access to the 1C: Education platform. Work was carried out on interactive whiteboards / multimedia projectors, stationary and mobile computer classes and students' home computers. The analysis of the results of testing presented in the electronic databases and the generalization of the results of testing presented in the reports of teachers, made it possible to clarify the diagnostic criteria for achieving the appropriate level of schoolchildren's success, to ensure the correspondence of the used task material of a search and research nature to the content of basic mathematical courses and to assess the effect of the adaptive learning systems on the dynamics of quantitative and qualitative changes in the profile of student learning success. Based on the results of the work, the following conclusions can be drawn:

The content and complexity level of the task material contained in the system corresponds to the current curriculum in mathematics for forms 2-3; the system contains a sufficient number of search and research tasks and the tasks are diverse in terms of difficulty - from the simplest to Olympiad level. The teachers who participated in the testing also noted the originality of the author's approach and the high quality methodology of the submitted tasks.

The system enables the objective identification of the individual character of the student’s level of success in mathematics and the problems that may affect the successful study of the subject: insufficient level of training, lack of motivation and low level of cognitive activity, insufficient mathematical intuition and of knowledge of heuristic techniques of mathematical activity. Moreover, the success profiles of schoolchildren obtained during testing generally coincided with the observations of teachers, which indicates the relative reliability of the proposed diagnostic tools.

The system enables the creation of favorable conditions for the improvement of the weaker components of student’s learning success, which can be traced to the change in the success profile when performing a set of tests on the topic (preliminary, formative and control). As a result of using the system, in a number of schools it was possible to increase the level of training, the level of cognitive activity and knowledge of heuristic techniques of educational mathematical activity for specific students on various topics of mathematics and to improve the results of the annual control analysis of mathematics performance in 2-3 forms, since many tasks in the control were related to the topics submitted for testing.

An interesting and unexpected result of testing the system was that in the course of working with it, the students' communicative and regulatory competencies developed along the way: when working with formative tests, students independently paired and discussed groups, discussed solutions, developed a common opinion, consulted with each other and with the teacher.

The system was fit for purpose. Among its advantages, teachers and students as test participants noted the following: ease of use; high speed of execution of operations for assigning tasks and verifying the completion of tasks; a visual representation of the results of the students' work, the ability to more flexibly and diversely measure the knowledge of students, to identify topics where improvement was needed and a subsequent ability to set an individual learning path to facilitate such improvement.
The participants in the testing also noted: an easy to use and understand interface; clear and accessible wording of assignments, which saves lesson time and helps the system assign individual tasks to students; an interactive mode of operation where, when mistakes are made, the student can see where and how they were made and how they can be fixed, providing the system with the opportunity to reinforce learning by presenting for completion tasks of a similar type and the ability to independently advance in a subject topic at an appropriate pace.

Teachers believed the system had generalized applicability to the overall educational process. In particular, it can be used in classroom activities, for issuing interactive homework and in extracurricular activities.

Thus, the results of testing in general can be considered positive. As shown by a survey of teachers participating in testing conducted online, the pedagogical and technological solutions developed as part of the study were shown to be effective and convenient to use. Among its advantages, teachers noted the ease of use and efficiency in assigning and verifying assignments; the ability to design and implement an individual educational and search trajectory for each student and a comprehensive visual representation of the quality of the student’s mathematical preparation in the form of a profile and success index. Most teachers expressed a desire to use this system in their professional activities in the next academic year and recommended it for more widespread distribution in schools.

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